Offer Curves

The offer curve is an alternative way to describe an individual's demand behavior, *i.e.*, his demand function. And by summing up individuals' demand behavior, we can also use the offer curve to describe the *market* demand function.

The offer curve is generally well-defined for any number of goods, but we want to focus on the two-good case for the strong geometric insight it provides in helping to understand the analytical concept. Throughout this note there will be only two goods, with quantities denoted by x and y.

The idea behind the offer curve is to depict the individual's demand behavior in the same space we use to depict his preferences (his indifference map) — namely, the commodity space, which, in the two-good case, is two-dimensional. You probably recall the *price-consumption curve* from your intermediate microeconomics course. There, as you change the price of one of the goods (say, the x-good) while holding constant the consumer's wealth and the price

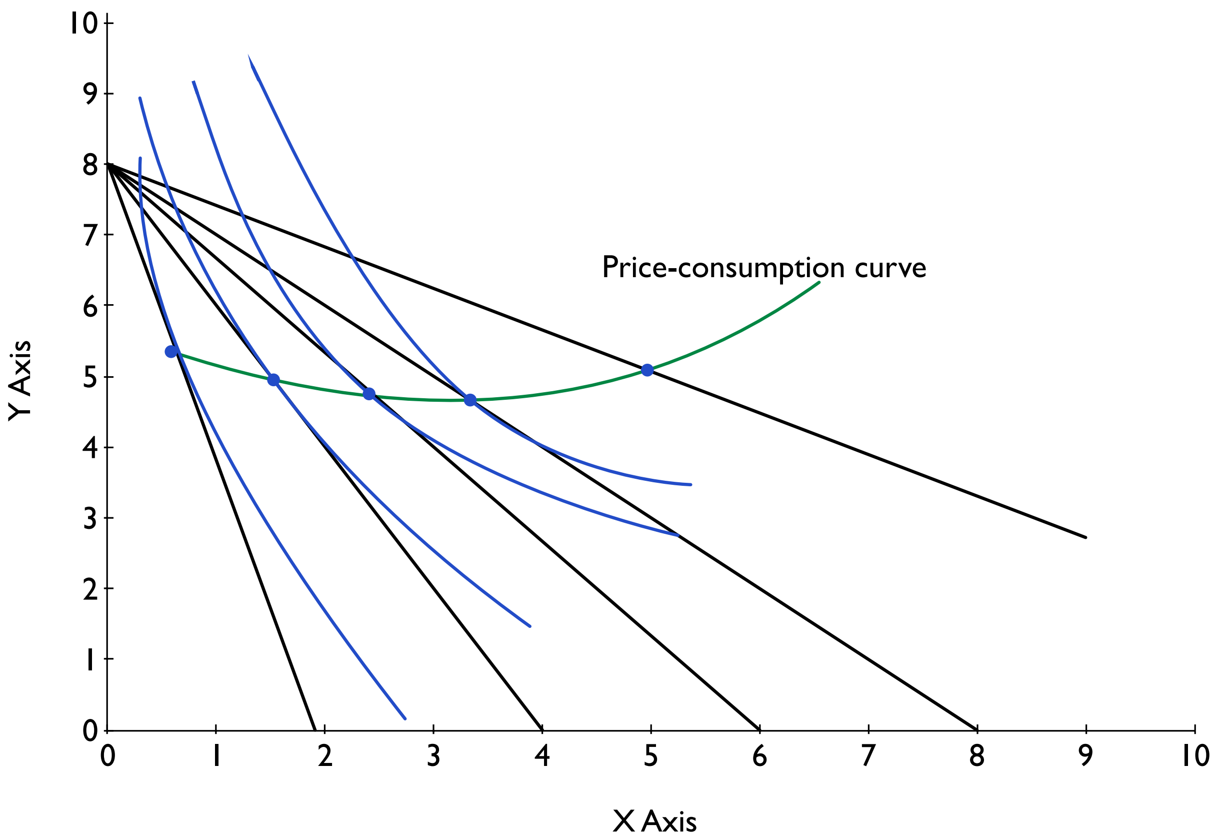


Figure 1: Price-consumption Curve

of the other good, you geometrically rotate the budget constraint and trace out the locus of bundles (x, y) the consumer would buy at the various prices p_x, as in Figure 1. Figure 2 retains the locus of (x, y)-bundles — the price-consumption curve — but omits the budget constraints and indifference curves. This price-consumption curve shows exactly the bundles this consumer would potentially purchase at the various possible prices p_x; all the other bundles in his commodity-space are ones he would *not* purchase, at any price (assuming the other good's price and the consumer's wealth, p_y and w, remain fixed at their original values).

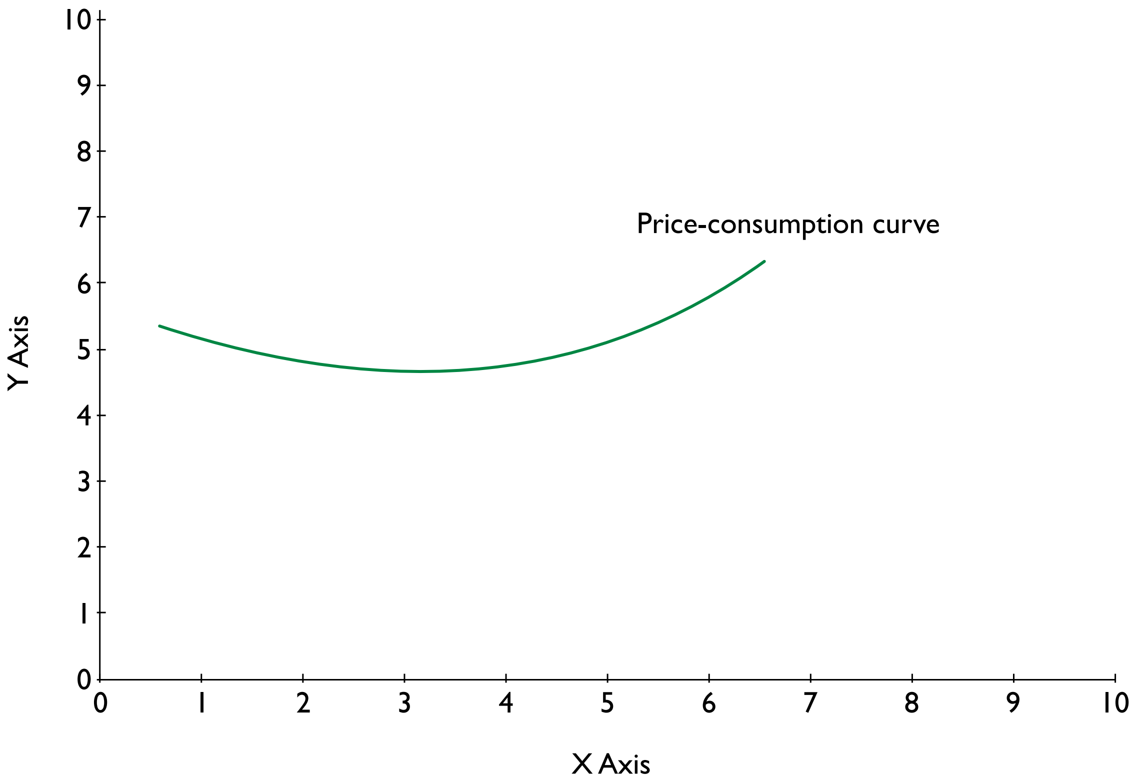


Figure 2: Price-consumption Curve

The **offer curve** is exactly the same concept, but in the general equilibrium context. So instead of holding constant the consumer's wealth or income, we hold constant his initial-endowment bundle $(\mathring{x}, \mathring{y})$. And instead of tracing out his chosen bundles (x, y) at all the various prices of just the x-good, the offer curve traces out the locus of *net* bundles $(\overset{\Delta}{x}, \overset{\Delta}{y})$ he would choose at all the various possible price-lists (p_x, p_y), or equivalently, at all the possible price-ratios (relative prices) $\rho = p_x/p_y$.

Definition: Let $\mathfrak{D}(\cdot) : \mathbb{R}^2_{++} \to \mathbb{R}^2$ be a net demand function: $\mathfrak{D}(p_x, p_y)$ gives the net bundle $(\overset{\Delta}{x}, \overset{\Delta}{y})$ as a function of the price-list (p_x, p_y). The **offer curve** for $\mathfrak{D}$ is the range of the function $\mathfrak{D}$ — *i.e.*, the set

$$\{\, (\overset{\Delta}{x}, \overset{\Delta}{y}) \in \mathbb{R}^2 \,|\, \mathfrak{D}(p_x, p_y) = (\overset{\Delta}{x}, \overset{\Delta}{y}) \text{ for some } (p_x, p_y) \in \mathbb{R}^2_{++} \}.$$

We often want to express an offer curve as a function — for example, $\overset{\Delta}{y} = h(\overset{\Delta}{x})$. The function h would provide the answer to the question "if the (net) quantity $\overset{\Delta}{x}$ of the x-good is chosen under $\mathfrak{D}$, how much of the y-good will be chosen?" The way to answer this question, and the way to derive the function h, is kind of a clever trick:

(1) For any quantity $\overset{\Delta}{x}$ of the x-good, we can determine from the demand function (actually, from the inverse of the demand function) what the price-ratio ρ would have to be in order for the amount $\overset{\Delta}{x}$ to be chosen.

(2) And then, at that price-ratio, we can use the budget constraint to determine the quantity $\overset{\Delta}{y}$ of the y-good that would be chosen: $\overset{\Delta}{y} = -\rho \, \overset{\Delta}{x}$.

If the demand function is single-valued and decreasing in a good's relative price, then the inverse demand function will be well-defined and we can carry out this operation.

Examples: The following pages contain some examples. First we derive the offer curve of one of the Cobb-Douglas consumers in our earlier Walrasian equilibrium numerical two-person, two-good example. Then we obtain the market net demand functions for both goods in that example, and we use the net demand functions first to simply trace out the market offer curve, and then to obtain a closed-form solution for the market offer curve (the function h described above, but for the *market* offer curve instead of just the individual's offer curve). Then there are two more examples in which offer curves are found for individual consumers.

INDIVIDUAL OFFER CURVES
IN OUR 2×2 NUMERICAL EXAMPLE

MR. 1: $u_1(x_1, y_1) = x_1^{7/8} y_1^{1/8}$, $(\mathring{x}_1, \mathring{y}_1) = (40, 80)$

$$x_1 = \frac{7}{8}\left(40 + \frac{80}{p}\right) = 35 + \frac{70}{p}, \quad y_1 = \frac{1}{8}(40p + 80) = 5p + 10$$

$\therefore\ \hat{x}_1 = -5 + \dfrac{70}{p}$, AND $p = \dfrac{70}{5 + \hat{x}_1}$ IS THE PRICE-RATIO

THAT WILL ELICIT THE NET DEMAND QUANTITY $\hat{x}_1$.

THE OFFER CURVE IS $\hat{y}_1 = -p\hat{x}_1 = -\dfrac{70\hat{x}_1}{5 + \hat{x}_1}$, GIVING

THE NET AMOUNT OF THE Y-GOOD THAT MR. 1 WILL DEMAND

IF HIS NET DEMAND FOR THE X-GOOD IS $\hat{x}_1$.

THUS, THE ONLY NET BUNDLES $(\hat{x}_1, \hat{y}_1)$ THAT MR. 1

COULD DEMAND ARE THE BUNDLES THAT SATISFY

THE EQUATION $\hat{y}_1 = -\dfrac{70\hat{x}_1}{5 + \hat{x}_1}$. ANY BUNDLE

$(\hat{x}_1, \hat{y}_1)$ THAT SATISFIES THIS EQUATION —

i.e., THAT LIES ON THE OFFER CURVE — WILL BE

THE BUNDLE ELICITED BY THE PRICE-RATIO $p = -\dfrac{\hat{y}_1}{\hat{x}_1}$.

MR. 2: $u_2(x_2, y_2) = x_2^{1/2} y_2^{1/2}$, $(\mathring{x}_2, \mathring{y}_2) = (80, 40)$.

$$x_2 = \frac{1}{2}\left(80 + \frac{40}{p}\right) = 40 + \frac{20}{p}, \quad y_2 = \frac{1}{2}(80p + 40) = 40p + 20.$$

$\therefore\ \hat{x}_2 = -40 + \dfrac{20}{p}$ AND $\therefore\ p = \dfrac{20}{40 + \hat{x}_2}$.

THE OFFER CURVE IS THEREFORE

$$\hat{y}_2 = -p\,\hat{x}_2 = -\frac{20\hat{x}_2}{40 + \hat{x}_2}.$$

THE MARKET OFFER CURVE

IN OUR NUMERICAL EXAMPLE

$$\left[\begin{aligned}(\mathring{x}_1,\mathring{y}_1)&=(40,80)\\ (\mathring{x}_2,\mathring{y}_2)&=(80,40)\\ \hline (\mathring{x},\mathring{y})&=(120,120)\end{aligned}\right]$$

INDIVIDUAL DEMAND FUNCTIONS:

$$x_1=\tfrac{7}{8}\left(40+\tfrac{80}{p}\right)=35+\tfrac{70}{p} \qquad y_1=\tfrac{1}{8}(40p+80)=5p+10$$

$$x_2=\tfrac{1}{2}\left(80+\tfrac{40}{p}\right)=40+\tfrac{20}{p} \qquad y_2=\tfrac{1}{2}(80p+40)=40p+20$$

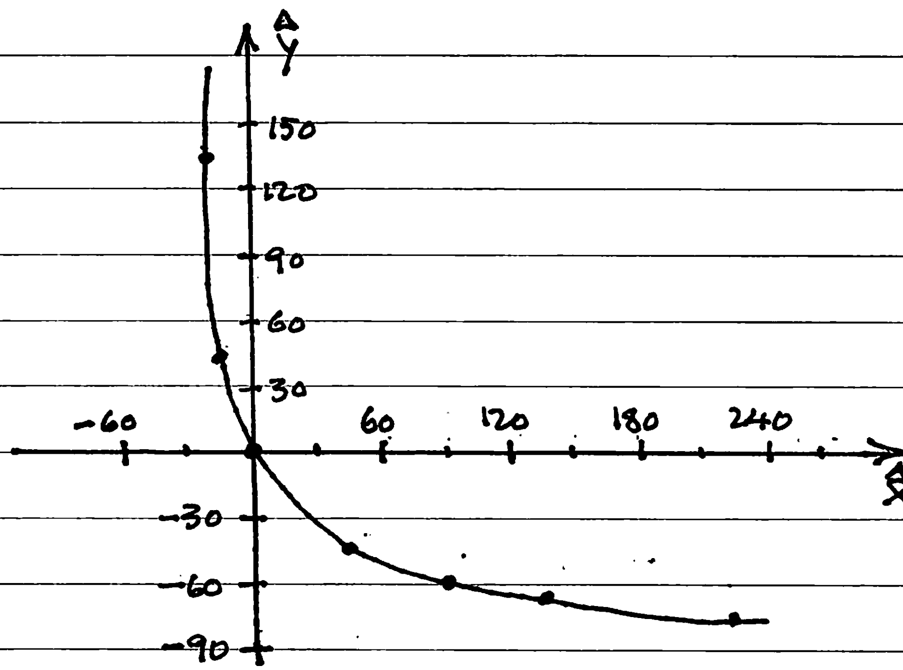
MARKET DEMAND: $X(p)=75+\frac{90}{p}$ $\qquad Y(p)=45p+30$

MARKET NET DEMAND: $\boxed{\overset{\Delta}{X}(p)=-45+\frac{90}{p},\quad \overset{\Delta}{Y}(p)=45p-90}$

[NOTE THAT $\overset{\Delta}{Y}(p)=-p\overset{\Delta}{X}(p)$, WALRAS' LAW HOLDS]

WITH THE MARKET NET DEMANDS IN HAND, WE'LL FIRST TRACE OUT THE MARKET (AGGREGATE) OFFER CURVE, AND THEN WE'LL OBTAIN A CLOSED-FORM SOLUTION FOR THE OFFER CURVE.

p	$\overset{\Delta}{X}(p)$	$\overset{\Delta}{Y}(p)$
1	45	−45
2	0	0
3	−15	45
5	−27	135
10	−36	360
2/3	90	−60
1/2	135	$-67\frac{1}{2}$
1/3	225	−75



A CLOSED-FORM EXPRESSION FOR THE MARKET OFFER CURVE IN THIS EXAMPLE:

[OBTAINING A FUNCTION $\overset{\Delta}{Y} = h(\overset{\Delta}{X})$]

(1) $\overset{\Delta}{X} = -45 + \frac{90}{p}$; i.e., $\overset{\Delta}{X} + 45 = \frac{90}{p}$

i.e., $p = \frac{90}{\overset{\Delta}{X}+45}$

THIS IS THE p THAT WILL ELICIT A MARKET NET DEMAND $\overset{\Delta}{X}$.

(2) $\overset{\Delta}{Y} = 45p - 90$

$= 45\left[\frac{90}{\overset{\Delta}{X}+45}\right] - 90$

$= 90\left[\frac{45}{\overset{\Delta}{X}+45} - \frac{\overset{\Delta}{X}+45}{\overset{\Delta}{X}+45}\right]$

$= -90\frac{\overset{\Delta}{X}}{\overset{\Delta}{X}+45}$

OFFER CURVE (EXAMPLE)

$u(x,y) = y + \beta x - \frac{1}{2}\alpha x^2 \qquad [\alpha \mathring{x} < \beta]$

$MRS = \beta - \alpha x; \therefore x = \frac{\beta}{\alpha} - \frac{1}{\alpha} p, \text{ IF } p \leqq \beta.$

$x = 0, \text{ IF } p \geqq \beta.$

$\overset{\Delta}{x} = x - \mathring{x} = \left(\frac{\beta}{\alpha} - \mathring{x}\right) - \frac{1}{\alpha} p \qquad$ NOTE: $\frac{\beta}{\alpha} - \mathring{x} > 0.$

$\frac{1}{\alpha} p = \frac{\beta}{\alpha} - \mathring{x} - \overset{\Delta}{x}.$

$p = (\beta - \alpha \mathring{x}) - \alpha \overset{\Delta}{x}$

$\overset{\Delta}{y} = -p \overset{\Delta}{x} = -(\beta - \alpha \mathring{x}) \overset{\Delta}{x} + \alpha \overset{\Delta}{x}^2.$

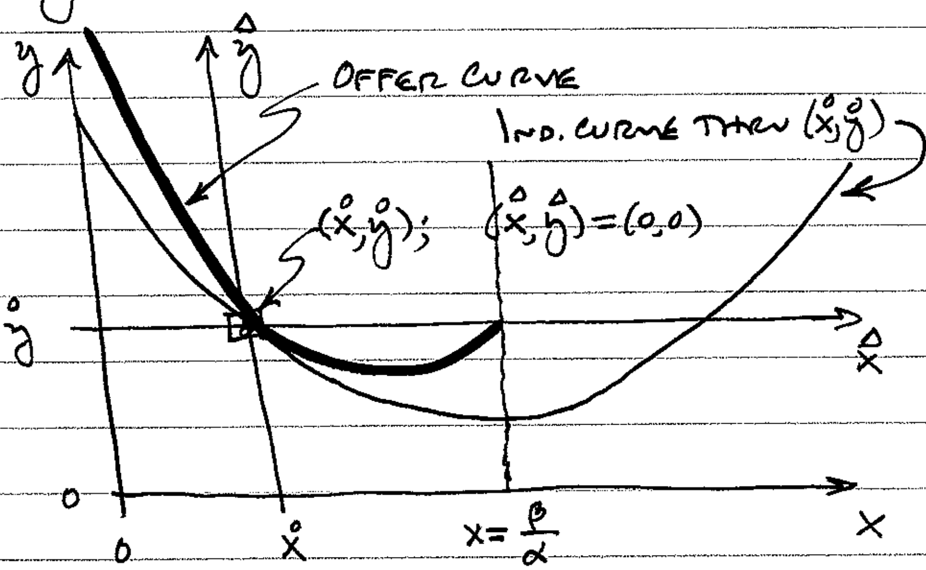
AS $p \to 0: \quad \overset{\Delta}{x} \to \frac{\beta}{\alpha} - \mathring{x}, \quad \overset{\Delta}{y} \to 0.$

IF $p = 0: \quad \overset{\Delta}{x} = \frac{\beta}{\alpha} - \mathring{x};$ i.e., $x = \frac{\beta}{\alpha}; \ \overset{\Delta}{y} = 0.$

~~[illegible]~~:

IF $p \geqq \beta: \quad x = 0, \ \overset{\Delta}{x} = -\mathring{x}, \ \overset{\Delta}{y} = -p\overset{\Delta}{x} = p\mathring{x}.$

$\therefore$ AS $p \to \infty, \ \overset{\Delta}{y} \to \infty.$



EXERCISE #85

OFFER CURVE (EXAMPLE)

$u(x,y) = y + \alpha \log x$

$MRS = \dfrac{\alpha}{x}$; $\therefore\ x = \dfrac{\alpha}{p}$, $\forall p > 0$.

$\overset{\Delta}{x} = x - \overset{\circ}{x} = \dfrac{\alpha}{p} - \overset{\circ}{x}$ $\quad \dfrac{\alpha}{p} = \overset{\Delta}{x} + \overset{\circ}{x}$ $\quad p = \dfrac{\alpha}{\overset{\circ}{x} + \overset{\Delta}{x}}$, $\overset{\Delta}{x} > -\overset{\circ}{x}$

$$\boxed{\overset{\Delta}{y} = -p\overset{\Delta}{x} = -\frac{\alpha \overset{\Delta}{x}}{\overset{\circ}{x} + \overset{\Delta}{x}}}, \quad \overset{\Delta}{x} > -\overset{\circ}{x}.$$

As $p \to 0$: $\overset{\Delta}{x} \to \infty$, $\overset{\Delta}{y} \to -\alpha$.

As $p \to \infty$: $\overset{\Delta}{x} \to -\overset{\circ}{x}$, $\overset{\Delta}{y} \to \infty$.