Adding Production to the Theory

We begin by considering the simplest situation that includes production: two goods, both of which have consumption value, but one of which can be transformed into the other. Production can go in only one direction: the input good can be used to produce the output good, but not vice versa. Some simple concrete examples are oranges as input to produce orange juice; grapes as input to produce wine; or leisure as input (where it's called labor) to produce "stuff".

Consumers: There are n consumers, indexed by $i \in N = \{1, \ldots, n\}$. Consumer i's consumption bundle is $(x_i, y_i) \in \mathbb{R}^2_+$, where x_i is his consumption of the input good and y_i is his consumption of the output good. Each consumer is described by a utility function $u^i : \mathbb{R}^2_+ \to \mathbb{R}$ and an initial bundle $(\mathring{x}_i, \mathring{y}_i) \in \mathbb{R}^2_+$.

Producers: There are m producers (also called **firms**), indexed by $j \in M = \{1, \ldots, m\}$. Producer j's production capabilities are described by a production function $f_j : \mathbb{R}_+ \to \mathbb{R}_+$: if producer j uses z_j units of the input good, then q_j units of the output good are produced, where $q_j = f_j(z_j)$.

The economy is therefore described by an n-tuple $\left(u^i, (\mathring{x}_i, \mathring{y}_i)\right)_1^n$ and an m-tuple $(f_j)_1^m$.

We begin by describing the feasible production-and-consumption plans, or **allocations**; then we'll describe the Pareto efficient allocations; and then we'll describe the Walrasian equilibria.

Feasible Allocations

An allocation is a $(2n+m)$-tuple $((x_i, y_i)_1^n, (z_j)_1^m) \in \mathbb{R}^{2n}_+ \times \mathbb{R}^m_+$. The feasible allocations are the ones that satisfy these two inequalities:

$$\begin{aligned} \sum_{i \in N} x_i + \sum_{j \in M} z_j &\leqq \mathring{x} \\ \sum_{i \in N} y_i &\leqq \mathring{y} + \sum_{j \in M} f_j(z_j). \end{aligned}$$

Pareto Allocations

We continue to define a Pareto improvement as an allocation in which at least one person is better off and no one is worse off. But what does it mean for a firm to be better off or worse off? When should we say that a firm prefers one production plan z_j, or (z_j, q_j), to another? The answer is that we *don't* say. We can't, for example, use profit to rank alternative plans, because prices are used in the definition of profit, and in our Pareto analysis we don't assume that markets and prices are what's used to arrive at allocations.

So we simply don't ascribe preferences to firms. We view the firms simply as machines, devices for transforming the input good into the output good. The firms thereby make it possible for the consumers to obtain allocations they wouldn't otherwise be able to obtain. And we continue to define welfare improvements in terms of only the consumers' welfare. Our definitions of Pareto improvements and Pareto efficiency are therefore the same as they were when no production was possible. They're defined only in terms of the consumers' preferences.

The Pareto allocations are therefore the solutions of a new version of the problem P-Max:

$$\max \lambda_1 u^1(x_1, y_1) \;\text{ s.t. }\; x_i, y_i, z_j \geqq 0 \;\; (\forall i \in N, j \in M),$$

$$\begin{aligned}
\sum_{i\in N} x_i + \sum_{j\in M} z_j &\leqq \mathring{x} && : \sigma_x \\
\sum_{i\in N} y_i &\leqq \mathring{y} + \sum_{j\in M} f_j(z_j) && : \sigma_y \\
u^i(x_i, y_i) &\geqq c_i \quad (i = 2, \ldots, n) && : \lambda_i.
\end{aligned}$$

The first-order marginal conditions for a solution of this problem are

$$\begin{array}{lll}
(1) & x_i: & \lambda_i u^i_x \leqq \sigma_x, \;\text{ with equality if } x_i > 0 \quad (\forall i \in N) \\
(2) & y_i: & \lambda_i u^i_y \leqq \sigma_y, \;\text{ with equality if } y_i > 0 \quad (\forall i \in N) \\
(3) & z_j: & 0 \leqq \sigma_x - \sigma_y f_j'(z_j), \;\text{ with equality if } z_j > 0 \quad (\forall j \in M)
\end{array}$$

In order to give an economic interpretation of the FOMC, let's simplify things by considering only solutions in which all the variables and Lagrange multipliers are strictly positive, so that all the FOMC are equations instead of inequalities. Then the conditions (1) and (2) are the familiar MRS conditions, $MRS^i = \dfrac{\sigma_x}{\sigma_y}$ for each $i \in N$, which yield the Equal-MRS condition

$$MRS^1 = MRS^2 = \cdots = MRS^n.$$

When we combine this with (3) we obtain

$$f_j'(z_j) = MRS^i_{xy} \;\text{ for all }\; i \in N, j \in M.$$

Writing MP^j for the marginal product of the input good for firm j, we can rewrite the above condition as

$$MP^j = MRS^i_{xy} \qquad \text{or as} \qquad \frac{dq_i}{dz_i} = -\frac{dy_i}{dx_i} \;\text{ for all }\; i \in N, j \in M,$$

— *i.e.*, the marginal value of the x-good in terms of the y-good, in both production and consumption, should be the same everywhere.

Equivalently, we could write the equations as

$$\frac{1}{f_j'(z_j)} = MRS^i_{yx} \qquad \text{or as} \qquad \frac{dz_i}{dq_i} = -\frac{dx_i}{dy_i} \;\; \text{for all} \;\; i \in N, j \in M.$$

The term on the left in each of the above equations is just the (real) marginal cost of the output good — how many units of the input good are required, at the margin, to produce an additional unit of output. So we could also write the equations as

$$MC^j = MRS^i_{yx} \qquad \text{or as} \qquad MC^j = MV^i \;\; \text{for all} \;\; i \in N, j \in M,$$

where MV^i is consumer i's marginal value (*i.e.*, marginal willingness to pay) for the output good, measured in units of the input good.

And of course the marginal conditions have to be supplemented by the constraint-satisfaction conditions

$$\sum_{i\in N} x_i + \sum_{j\in M} z_j = \mathring{x} \qquad \text{and} \qquad \sum_{i\in N} y_i = \mathring{y} + \sum_{j\in M} f_j(z_j).$$

(These are equations instead of inequalities because we assumed that $\sigma_x, \sigma_y > 0$.)

Robinson Crusoe (A Single Consumer):

Let's apply our Pareto analysis to the simplest case that includes production: Robinson Crusoe. Robinson is alone on a desert island (oddly, he looks a lot like Tom Hanks). So he's the only consumer. Let's assume there's a single production process (called "climbing palm trees"), which can turn Robinson's leisure/labor into coconuts. We have $n = m = 1$, and in this simple case we can represent Robinson's preferences and his production technology geometrically as in Figure 1, and we can represent them both together, as in Figure 2.

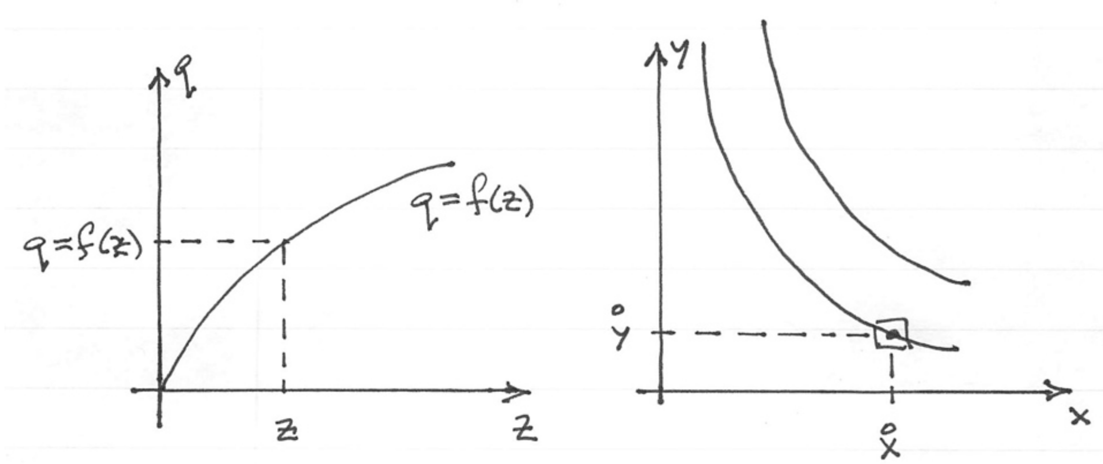


Figure 1

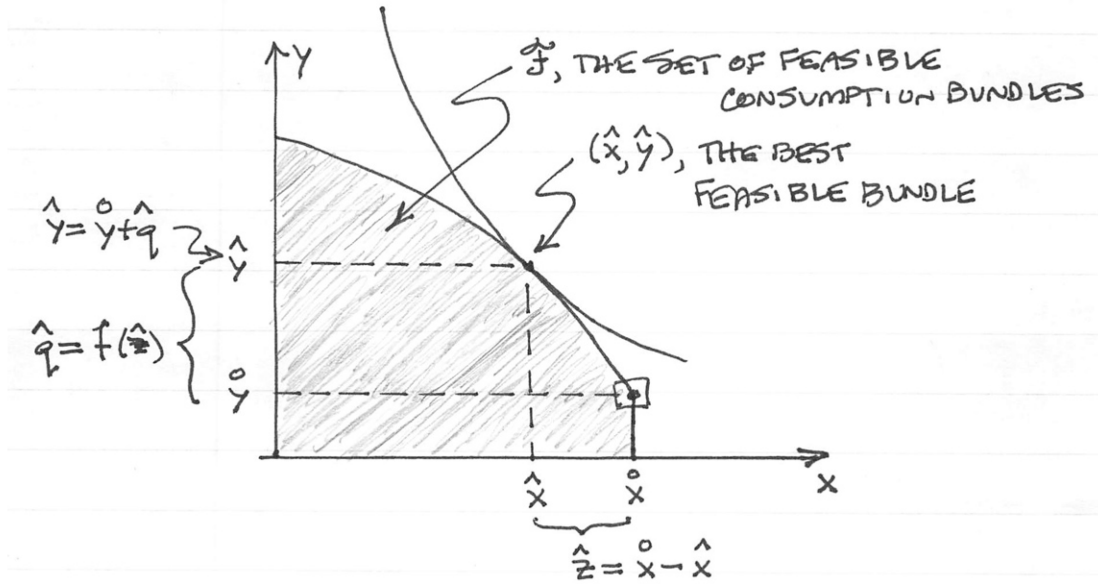


Figure 2

The Pareto maximization problem is simply Robinson's utility-maximization problem. He's not maximizing utility subject to a budget constraint, but subject to the two constraints in the P-max problem, $x + z \leqq \mathring{x}$ and $y \leqq \mathring{y} + f(z)$. Together, these two constraints yield the consumption-possibilities set $\mathcal{F}$ in Figure 2. Assuming that Robinson's utility function is strictly increasing, any solution will satisfy both inequalities as equations. Converting the inequalities to equations, we can write the problem as an unconstrained maximization problem with a single variable, z (this is not necessary, it's just just a trick we often use to simplify things):

$$\max_z u\big(\mathring{x} - z, \mathring{y} + f(z)\big).$$

The first-order condition for an interior solution is

$$\frac{d}{dz} u\big(\mathring{x} - z, \mathring{y} + f(z)\big) = 0 \;;$$

$$i.e., \qquad -u_x + f'(z) u_y = 0 \;;$$

$$i.e., \qquad f'(z) = \frac{u_x}{u_y} \;;$$

$$i.e., \qquad MP = MRS \;.$$

Fisher's Separation Theorem:

Now suppose that Robinson would like to decentralize the allocation that maximizes his utility, separating the production decision from the consumption decision. After his man Friday appears (or is it Wilson?), he might even arrange to have the decisions made by different people — Wilson choosing the production plan and Robinson himself choosing the consumption bundle. It seems as if he could do that by using markets and prices. He needs to find prices p_x and p_y at which

- as production manager, Robinson (or his man Wilson) will be led to choose the right z by maximizing profit, and
- as consumption manager ("buyer"), Robinson will be led to choose the right bundle (x, y) by maximizing $u(\cdot)$ subject to the budget constraint defined by the prices p_x and p_y .

"Fisher's Separation Theorem" says that separating production decisions from consumption decisions in this way, using markets and prices, will yield the same level of welfare as if we centralized the decision. Nothing is lost by decentralizing/separating, the decisions. (Irving Fisher actually stated his "theorem" for investment and portfolio decisions being made separately from the consumption decisions that are made by the owners of firms or portfolios. And it's not really a theorem in the technical sense. But in our context here, it's really just the First Welfare Theorem, for the Walrasian model when it includes production.)

The significance of Fisher's separation concept for doing microeconomics is that even if an individual is making production/investment decisions and also consumption decisions, we model that individual as if he were two separate decision-makers — a producer, or firm, on the one hand, and a consumer on the other hand. Even if Robinson *doesn't* have Wilson to make his production decisions, we still model Robinson separately as a firm and as a consumer.

But does this work? Does the First Welfare Theorem still hold when the model includes production?

Accounting for Profit

In order to elicit the right production plan z by using prices p_x and p_y and directing his firm to maximize profit, Robinson will obviously have to choose prices that satisfy

$$\frac{p_x}{p_y} = \frac{\sigma_x}{\sigma_y} = f'(z),$$

where σ_x and σ_y are the Lagrange multiplier values at the solution of the P-max problem. But as Figure 3 suggests, the budget constraint with these prices,

$$p_x x + p_y y \leqq p_x \mathring{x} + p_y \mathring{y},$$

might not allow the consumer-Robinson to choose a bundle (x, y) as large as the one that maximizes Robinson's utility among all feasible bundles. This is because the *average* rate of transformation in Figure 3 is larger than the *marginal* rate of transformation. There's a "surplus," a positive profit.

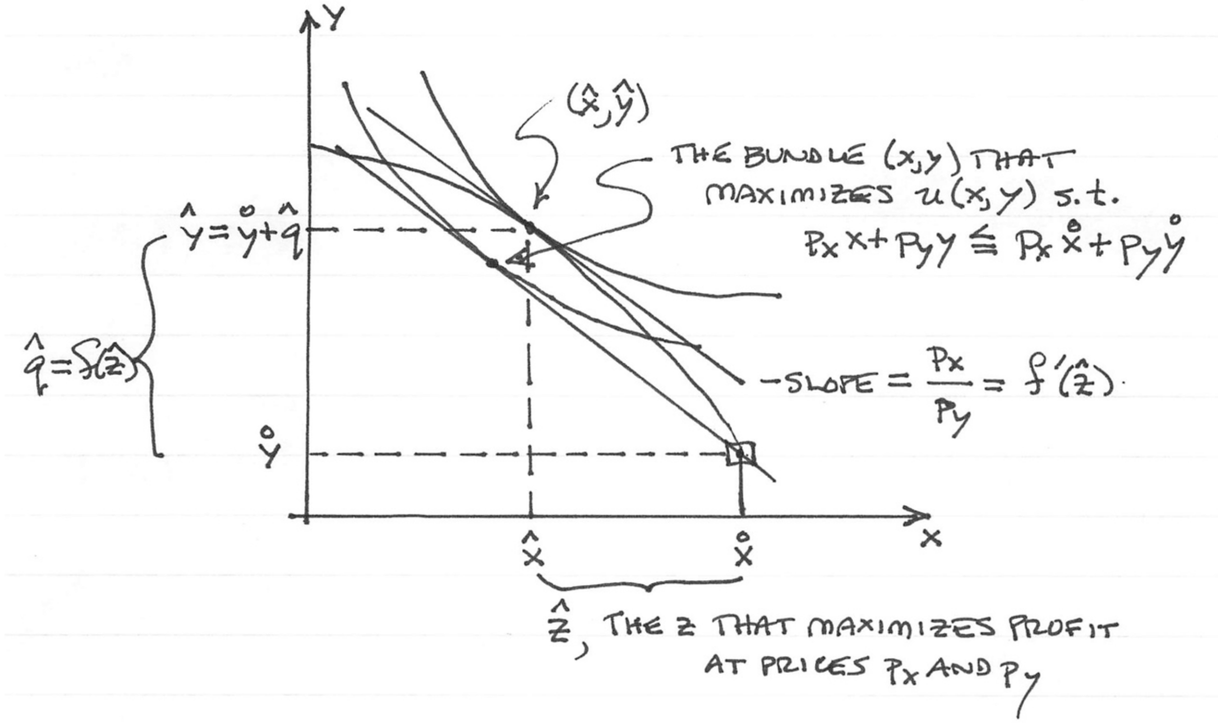


Figure 3

Let's return to the case where n and m can be greater than 1, and try writing down a definition of Walrasian equilibrium when there's production.

Tentative Definition: A Walrasian equilibrium is a triple

$$\big((\widehat{p}_x, \widehat{p}_y), (\widehat{z})_1^m, (\widehat{x}_i, \widehat{y}_i)_1^n\big) \in \mathbb{R}_+^2 \times \mathbb{R}_+^m \times \mathbb{R}_+^{2n}$$

that satisfies

(U-max) $\quad \forall i \in N: \ (\widehat{x}_i, \widehat{y}_i)$ maximizes u^i subject to

$$\text{(BC)} \quad \widehat{p}_x x_i + \widehat{p}_y y_i \leqq \widehat{p}_x \mathring{x}_i + \widehat{p}_y \mathring{y}_i$$

(π-max) $\quad \forall j \in M: \ \widehat{z}_j$ maximizes $\pi_j(z_j) := \widehat{p}_y f_j(z_j) - \widehat{p}_x z_j$

(M-clr)

$\sum_1^n \widehat{x}_i + \sum_1^m \widehat{z}_j \leqq \sum_1^n \mathring{x}_i$, with equality if $\widehat{p}_x > 0$

$\sum_1^n \widehat{y}_i \leqq \sum_1^n \mathring{y}_i + \sum_1^m f_j(\widehat{z}_j)$, with equality if $\widehat{p}_y > 0$.

Note that for any prices p_x and p_y, any allocation that satisfies the budget constraints (BC) for each consumer $i \in N$ will satisfy

$$p_x \left[\sum x_i - \sum \mathring{x}_i\right] + p_y \left[\sum y_i - \sum \mathring{y}_i\right] \leqq 0.$$

When there's no production, this is just Walras' Law.

If total profits happen to be positive, as in Figure 3, *i.e.*, if

$$\sum_{j=1}^{m} \Big[p_y f_j(z_j) - p_x z_j\Big] > 0 \,,$$

then we have

$$p_x \Big[\sum x_i - \sum \mathring{x}_i\Big] + p_y \Big[\sum y_i - \sum \mathring{y}_i\Big] - \Big[p_y \sum f_j(z_j) - p_x \sum z_j\Big] < 0$$

i.e.,

$$(*) \qquad p_x \Big[\sum x_i + \sum z_j - \sum \mathring{x}_i\Big] + p_y \Big[\sum y_i - \big(\sum \mathring{y}_i + \sum f_j(z_j)\big)\Big] < 0 \,.$$

If we write

$$E_x := \sum x_i + \sum z_j - \sum \mathring{x}_i \quad \text{ and } \quad E_y := \sum y_i - \big(\sum \mathring{y}_i + \sum f_j(z_j)\big)$$

for the amounts by which the allocation $\big((x_i, y_i)_{i\in N}, (z_j)_{j\in M}\big)$ exceeds the available resources, then $(*)$ is the inequality

$$(**) \qquad p_x E_x + p_y E_y < 0 \,.$$

This has two implications for Walrasian equilibrium and Pareto efficiency:

- An allocation that satisfies $(**)$ can't be Pareto efficient with efficiency prices (σ_x, σ_y) proportional to (p_x, p_y) because Pareto efficiency requires that $\sigma_x E_x + \sigma_y E_y = 0$ (by summing the two Constraint Satisfaction conditions for Pareto efficiency). Therefore, a Walrasian equilibrium can't be Pareto efficient.

- A price-list (p_x, p_y) and an allocation $\big((x_i, y_i)_1^n, (z_j)_1^m\big)$ that satisfy $(**)$ can't be a Walrasian equilibrium (according to the tentative definition we've given), because they can't satisfy both of the (M-Clr) market-clearing conditions in the definition: the two conditions together imply that $p_x E_x + p_y E_y = 0$. Therefore a Walrasian equilibrium can't exist.

The first implication is certainly not good news. But the second implication suggests that there may be something wrong with our tentative definition of Walrasian equilibrium. The problem clearly involves profit: when it's positive, it seems to cause a violation of Walras' Law.

Recall that the feasible allocations are the ones that satisfy the two inequalities

$$\sum_{i\in N} x_i + \sum_{j\in M} z_j \;\leqq\; \mathring{x} \quad \text{ and } \quad \sum_{i\in N} y_i \;\leqq\; \mathring{y} + \sum_{j\in M} f_j(z_j) \,.$$

Writing these as equations (*i.e.*, assuming that both goods are fully utilized), and writing the sums as simply $x, y, \mathring{x}, \mathring{y}, z, q$, we have

$$x + z = \mathring{x} \qquad \text{and} \qquad y = \mathring{y} + q\,.$$

These are simply accounting identities for the two goods. For *any* prices p_x and p_y, then, we have

$$p_x x + p_x z = p_x \mathring{x} \qquad \text{and} \qquad p_y y = p_y \mathring{y} + p_y q\,.$$

Combining the two equations yields

$$p_x x + p_y y = p_x \mathring{x} + p_y \mathring{y} \;+\; \big(p_y q - p_x z\big)\,;$$

i.e.,

Value of consumption $=$ Value of the endowment $+$ Net value of production activity.

We've converted the accounting identities for *quantities* of the two goods into a single accounting identity for the *value* of the goods — which is Walras' Law when there's production. If profit, $p_y q - p_x z$, is positive, then the total value of consumers' endowment is not large enough, by itself, to pay for the entire value of the goods available after production has taken place. Total consumption (x, y) will satisfy either $x < \mathring{x} - z$ or $y < \mathring{y} + q$, or both, as in Figure 3. Consumers' income, $p_x \mathring{x} + p_y \mathring{y}$, therefore has to be augmented by the firms' profits: we have to account for the distribution of profits (the "surplus value") arising from production in our Walrasian model of allocation via markets and prices.

We typically account for profit by assuming it all goes to consumers. In the Walrasian model we assume that consumers own shares of the firms' profits: θ_{ij} denotes consumer i's share of firm j's profit (with $0 \leqq \theta_{ij} \leqq 1$ for all i and j, and $\sum_{i \in N} \theta_{ij} = 1$ for each $j \in M$) .

We alter the tentative definition of Walrasian equilibrium we gave earlier by changing the budget constraint of each consumer $i \in N$ to

$$\widehat{p}_x x_i + \widehat{p}_y y_i \leqq \widehat{p}_x \mathring{x}_i + \widehat{p}_y \mathring{y}_i + \sum_{j \in M} \theta_{ij} \pi_j(\widehat{z}_j)\,,$$

where $\pi_j(z_j) := \widehat{p}_y f_j(z_j) - \widehat{p}_x z_j$ is firm j's profit as a function of its production plan z_j.

If every firm's f_j is continuous and concave, then with this new definition of equilibrium for our one-input one-output model with production, it's straightforward to use the same methods we've used before, when there was no production, to establish the existence of an equilibrium as well as the two welfare theorems.

For the more general case, with ℓ goods, a firm may have many inputs and many outputs, and a particular good might be an input for some firms and an output for others. We can no longer represent a firm's production technology (its feasible production plans) by a production function, because a given input vector will in general yield many different possible output vectors. Instead, we represent a firm's technology by a **production set** $Y_j \subseteq \mathbb{R}^\ell$: Y_j is the set of all production plans (input-output $\ell-$tuples $\mathbf{y}_j$) that the firm is able to carry out, and the firm chooses one of the plans $\mathbf{y}_j \in Y_j$. We use the convention that $y_{jk} > 0$ means that firm j is (net) *producing* good k as an output, and $y_{jk} < 0$ means that firm j is (net) *using* good k as an input. Consequently, firm j's profit from a production plan $\mathbf{y}_j$ is its revenue from its outputs minus the cost of its inputs:

$$\pi_j(\mathbf{y}_j) := \mathbf{p} \cdot \mathbf{y}_j = \sum_{k \in \mathcal{O}} p_k y_{jk} + \sum_{k \in \mathcal{I}} p_k y_{jk} = \sum_{k \in \mathcal{O}} p_k y_{jk} - \sum_{k \in \mathcal{I}} p_k(-y_{jk}) \, ,$$

where $\mathcal{O}$ is the set of goods that appear as outputs in $\mathbf{y}_j$ and $\mathcal{I}$ is the set of goods that appear as inputs in $\mathbf{y}_j$. (Since $\mathcal{O}$ and $\mathcal{I}$ depend upon $\mathbf{y}_j$, I should actually write $\mathcal{O}(\mathbf{y}_j)$ and $\mathcal{I}(\mathbf{y}_|)$.)

This gives us the following two definitions:

Definition: A **private ownership economy** is a list $E = \big((u_i, \mathring{\mathbf{x}}_i)_{i \in N}, (Y_j)_{j \in M}, (\theta_{ij})_{N \times M}\big)$ where for all $i \in N$ and all $j \in M$,

$$u_i : \mathbb{R}^\ell_+ \to \mathbb{R}, \quad \mathring{\mathbf{x}}_i \in \mathbb{R}^\ell_+, \quad Y_j \subseteq \mathbb{R}^\ell_+, \quad 0 \leqq \theta_{ij} \leqq 1, \text{ and } \sum_{i' \in N} \theta_{i'j} = 1 \, .$$

Definition: A **Walrasian equilibrium** of an economy $E = \big((u^i, \mathring{\mathbf{x}}^i)_{i \in N}, (Y_j)_{j \in M}, (\theta_{ij})_{N \times M}\big)$ is a triple $\big(\widehat{\mathbf{p}}, (\widehat{\mathbf{x}}^i)_N, (\widehat{\mathbf{y}}^j)_M\big) \in \mathbb{R}^\ell_+ \times \mathbb{R}^{n\ell}_+ \times \mathbb{R}^{m\ell}$ that satisfies

(U-max) $\quad \forall i \in N :$ $\widehat{\mathbf{x}}^i$ maximizes u^i subject to $\widehat{\mathbf{p}} \cdot \mathbf{x}^i \leqq \widehat{\mathbf{p}} \cdot \mathring{\mathbf{x}}^i + \sum_{j \in M} \theta_{ij} \pi(\mathbf{y}^j)$

(π-max) $\quad \forall j \in M :$ $\widehat{\mathbf{y}}^j$ maximizes $\pi(\mathbf{y}^j) := \widehat{\mathbf{p}} \cdot \mathbf{y}^j$ subject to $\mathbf{y}_j \in Y_j$

(M-clr) $\quad$ For $k = 1, \ldots, \ell$: $\sum_1^n \widehat{\mathbf{x}}^i_k \leqq \sum_1^n \mathring{\mathbf{x}}^i_k + \sum_1^m \widehat{\mathbf{y}}^j_k$, with equality if $\widehat{p}_k > 0$.

What about the existence and Pareto efficiency of equilibrium? If each consumer's preference is locally nonsatiated, our proof of the First Welfare Theorem can be straightforwardly augmented with producers and still goes through. In the one-input-one-output case, if the consumers and firms all have well-defined and continuous demand and supply functions over all price-lists $\mathbf{p} \in \mathbb{R}^2_+$:

- We don't need to make any change at all in our two-good proof of existence of equilibrium; we simply need to recognize that the market excess demand function will now be the sum of consumers' demand functions, minus the firms' supply functions.

- If the functions are all differentiable, then we can use the same calculus proofs of the two welfare theorems as we've already given, augmenting them with firms.

Of course, this still leaves open the issues of existence and the Second Welfare Theorem if there are more than two goods, or if the behavioral functions may not be single-valued or may not be defined when some prices are zero. These are issues we'll take up later in the course, after we've developed the necessary mathematical tools.

Constant Returns to Scale

Scale properties of production sets (firms' technological capabilities) are important. Here we address only constant returns to scale.

Definition: A production set Y has **constant returns to scale** if it satisfies the condition

$$\text{if } \mathbf{y} \in Y \text{ and } \alpha \in \mathbb{R}_+, \text{ then } \alpha\mathbf{y} \in Y.$$

Suppose $\mathbf{p}$ is the prevailing market price-list. If a firm has constant returns to scale and there is any production plan $\mathbf{y}$ in its production set Y that yields positive profit $\mathbf{p} \cdot \mathbf{y}$ at these prices, then every scaled-up plan $\alpha\mathbf{y}$ for any $\alpha > 1$ will be feasible for the firm and will yield a larger profit, $\alpha\mathbf{p} \cdot \mathbf{y}$. Therefore this firm does not have any profit-maximizing production plan at these prices — the firm cannot satisfy the profit-maximization hypothesis. However, constant returns to scale guarantees that the plan $\mathbf{0}$ — engaging in no productive activity — is in Y, and the plan $\mathbf{0}$ yields zero profit. Combining these two facts tells us that if a firm with constant returns to scale does have a profit-maximizing production plan $\mathbf{y}$, then

- the firm's profit is zero, and
- every scaled-up or scaled-down plan $\alpha\mathbf{y}$ is also profit-maximizing — and therefore a constant-returns-to-scale firm will not have a unique profit-maximizing production plan.

For a firm with constant returns to scale, then, we needn't be concerned about the distribution of its profit as we were above.

EXAMPLE:

$$f(z) = \begin{cases} 40 \log z, & z \geq 1 \\ 0, & z \leq 1 \end{cases} \qquad u(x,y) = y + 80 \log x \qquad (\mathring{x}, \mathring{y}) = (60, 40)$$

$$f'(z) = \frac{40}{z} \text{ IF } z > 1 \qquad MRS = \frac{80}{x}$$

PARETO FOC:

$f'(z) = MRS$ (INTERIOR)

i.e., $\frac{40}{z} = \frac{80}{x}$

$40x = 80z$

$40(\mathring{x} - z) = 80z$

$120z = 40\mathring{x}$

$z = \frac{1}{3}\mathring{x} = 20$

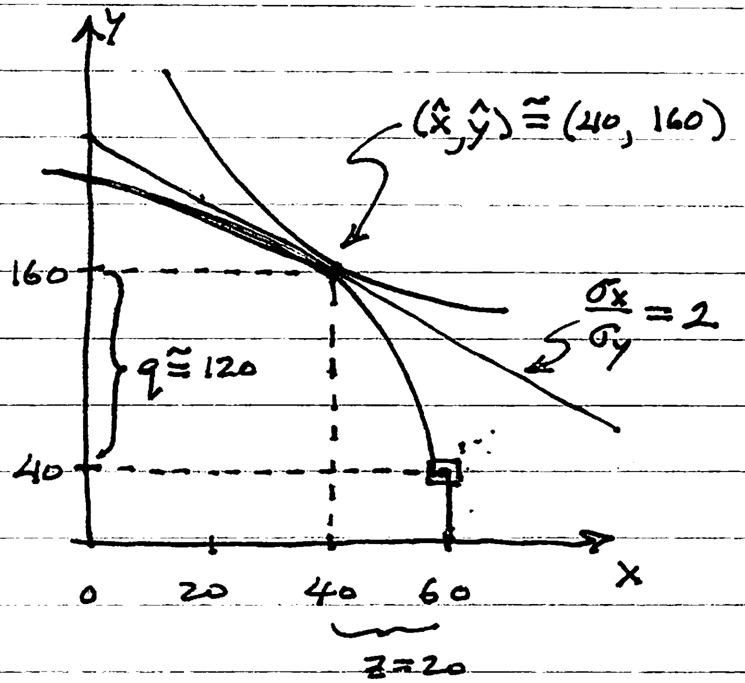
$\therefore q = f(z) = 40 \log 20$

$\cong (40)(3.00)$

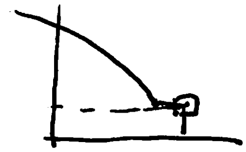
$= 120$

$\therefore x = \mathring{x} - z = 40$

$y = \mathring{y} + q \cong 160$



$$\left. \begin{aligned} f'(20) &= \frac{40}{20} = 2 \\ MRS &= \frac{80}{40} = 2 \end{aligned} \right\} \text{EFFICIENCY PRICES SATISFY } \frac{P_x}{P_y} = 2.$$



$$f(z) = \begin{cases} \alpha \log z, & \text{IF } z \geqq 1 \\ 0, & \text{IF } z \leqq 1 \end{cases}$$

EXAMPLE:

$f(z) = \alpha \log z$ $\qquad u(x,y) = x + 2y$ $\qquad (\mathring{x}, \mathring{y}) = (60, 40).$

$\therefore f'(z) = \frac{\alpha}{z}$ $\qquad \therefore MRS = \frac{1}{2}, \forall (x,y)$

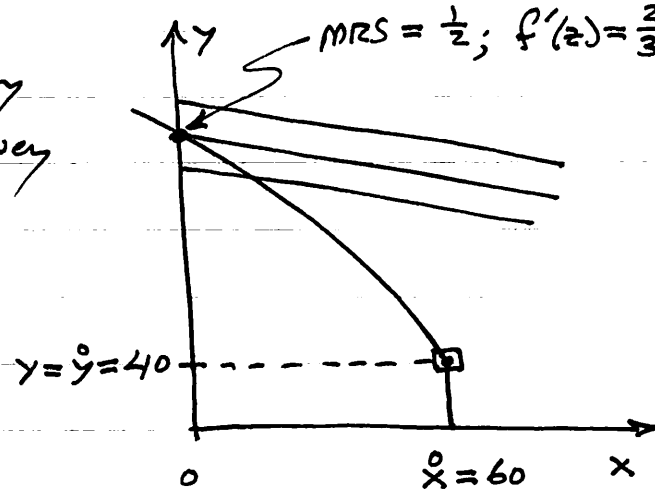
LET $\alpha = 40$.

NOTE THAT $f'(60) = \frac{40}{60} = \frac{2}{3} > MRS.$

$\therefore$ WE SHOULD SHIFT MORE OF THE X-GOOD INTO THE PRODUCTION PROCESS AS INPUT. BUT AT $z = 60$ WE HAVE $z = \mathring{x}$ — NO MORE OF THE X-GOOD IS AVAILABLE FOR INPUT.

$\therefore$ THE SOLUTION $z = 60$, $x = 0$, $y = 40 + 40 \log 60$ $\cong 204$ IS THE UNIQUE EFFICIENT OUTCOME.

WHAT ABOUT EFFICIENCY PRICES? THE ONLY EFFICIENCY PRICES ARE THOSE THAT SATISFY $\frac{p_x}{p_y} = \frac{2}{3}$.



IT MIGHT SEEM FROM THE DIAGRAM THAT ANY (p_x, p_y) SATISFYING $\frac{1}{2} \leqq \frac{p_x}{p_y} \leqq \frac{2}{3}$ WOULD DO. BUT IF $\frac{p_x}{p_y} < \frac{2}{3}$ — i.e., $\frac{p_x}{p_y} < MP$ — THEN THE FIRM WOULD CHOOSE TO INCREASE ITS PRODUCTION LEVEL (i.e., INCREASE z, BEYOND $z = 60$). OF COURSE, THE SUPPLY OF THE X-GOOD WOULD BE INSUFFICIENT TO CLEAR THE MARKET: IT WOULD NOT BE AN EQUILIBRIUM.