CORE ALLOCATIONS ARE NEARLY COMPETITIVE
IF THE ECONOMY IS LARGE:
FORMULATING THE IDEA

"IF THE ECONOMY IS LARGE, THEN A CORE ALLOCATION IS VERY NEARLY A COMPETITIVE ALLOCATION."

$\longrightarrow$ HOW LARGE? HOW NEAR?

ANALOGY:

"IF n IS A LARGE NUMBER, THEN $\frac{1}{n}$ IS VERY NEARLY ZERO."

DEF'N: IF THE NUMBER $a \in \mathbb{R}$ SATISFIES

$$\forall \varepsilon > 0 : \exists \hat{n} : n > \hat{n} \Rightarrow |x_n - a| < \varepsilon,$$

THEN WE SAY THAT a IS THE LIMIT OF THE SEQUENCE $\{x_n\}$, WHICH WE WRITE $\lim_{n \to \infty} x_n = a$.

"AS ~~THE ECONOMY~~ WE CONSIDER LARGER ECONOMIES, THE SET OF CORE ALLOCATIONS CONVERGES TO THE SET OF COMPETITIVE ALLOCATIONS."

DEF'NS:

(1) LET $B \subseteq \mathbb{R}^\ell$ AND $x \in \mathbb{R}^\ell$; $d(x, B) =_{def} \inf_{y \in B} \|x - y\|$.

(2) LET $A, B \subseteq \mathbb{R}^\ell$; $d(A, B) =_{def} \sup_{x \in A} d(x, B)$.

(3) $\lim_{n \to \infty} A_n = B$ IF $\lim_{n \to \infty} d(A_n, B) = 0$.

"If $\{E(n)\}$ is a sequence of economies in which, for each n, $E(n)$ has n consumers and $W(n)$ and $C(n)$ denote the sets of Walras and core allocations, then $\lim_{n \to \infty} d(W(n) - C(n)) = 0$."

DIFFICULTIES:

① As n changes, $C(n)$ and $W(n)$ lie in different spaces: $C(n), W(n) \subseteq \mathbb{R}^{n\ell}$.

② Conclusion may be untrue if $\{E(n)\}$ itself is not (in some sense) converging. Suppose, for example, that
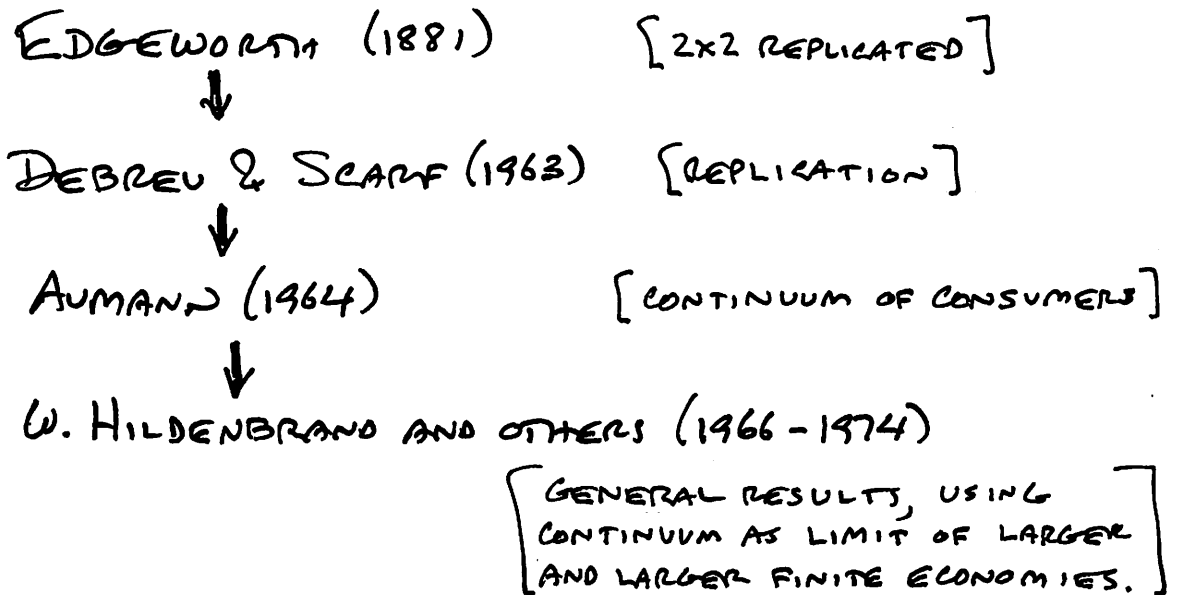
$$\overset{\circ}{x}{}^i(n) = \begin{Bmatrix} (0, n) & \text{if } i \text{ odd} \\ (n, 0) & \text{if } i \text{ even} \end{Bmatrix}; \quad u^i(n) = x_1^i x_2^i, \quad \forall i, n.$$

"If $\{E(n)\}$ converges and $\lim_{n \to \infty} W(n) = W$, then $\lim_{n \to \infty} C(n) = W$ as well."

... requires a def'n of distance between alternative preferences, and we again face difficulty ①.

It is becoming clear that this whole idea is very difficult to formulate clearly and rigorously — and, once that is done, it will probably be difficult to prove the desired theorem.

THE DEVELOPMENT OF THE IDEA:

Edgeworth (1881)   [2x2 replicated]

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Debreu & Scarf (1963)   [replication]

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Aumann (1964)   [continuum of consumers]

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W. Hildenbrand and others (1966-1974)

[General results, using continuum as limit of larger and larger finite economies.]

Later work (1975-1990): Brown, Khan, Anderson.