THE ARROW-DEBREU CONTINGENT CLAIMS MODEL:
AN INTERPRETATION OF THE WALRASIAN GE MODEL

IF $S=1$ IT'S OUR INTERTEMPORAL MODEL

	THE BASIC GE MODEL	THE ARROW-DEBREU MODEL
BUNDLES, CONSUMPTION PLANS	$x \in \mathbb{R}^{\ell}_{+}$	$(x_0, (x_s)_{s\in S}) \in \mathbb{R}^{C}_{+} \times \mathbb{R}^{SC}_{+} = \mathbb{R}^{(1+S)C}_{+}$
PREFERENCE	$\succsim$ ON $\mathbb{R}^{\ell}_{+}$ OR $u: \mathbb{R}^{\ell}_{+} \to \mathbb{R}$	$\succsim$ ON $\mathbb{R}^{(1+S)C}_{+}$ OR $u: \mathbb{R}^{(1+S)C}_{+} \to \mathbb{R}$
PRICES	$p \in \mathbb{R}^{\ell}_{+}$	$(p_0, (p_s)_{s\in S}) \in \mathbb{R}^{(1+S)C}_{+}$
BUDGET CONSTRAINT	$p\cdot x \leqq p \cdot \mathring{x}$	$p_0\cdot x_0 + \sum_{s\in S} \underbrace{p_s\cdot x_s}_{=\sum_{c\in C} p_{sc}x_{sc}} \leqq p_0\cdot \mathring{x}_0 + \sum_{s\in S} p_s\cdot \mathring{x}_s$

FOR THE MOST PART WE'RE GOING TO KEEP THINGS SIMPLE BY SETTING $C=1$:

PLAN: $(x_0, (x_s)_S) \in \mathbb{R}^{1+S}_{+}$ — IN OUR EXTENDED EXAMPLE: $S=\{H,L\}$, SO $1+S=3$

UTILITY FUNCTION: $u: \mathbb{R}^{1+S}_{+} \to \mathbb{R}$

BUDGET CONSTRAINT: $p_0 x_0 + \sum_{s\in S} p_s x_s \leqq p_0 \mathring{x}_s + \sum_{s\in S} p_s \mathring{x}_s$

NOT DOT PRODUCTS

⊛ → THE DEFINITIONS AND THEOREMS FROM OUR BASIC GE MODEL STILL APPLY HERE.

INTERPRETATION:

x_{sc} IS THE QUANTITY OF COMMODITY c ONE IS CONTRACTING TO HAVE DELIVERED TO HIM (AT A GIVEN TIME IN THE FUTURE) IF AND ONLY IF STATE s OCCURS. DELIVERY IS "CONTINGENT" ON STATE s OCCURRING, AND $(x_{sc})_{s\in S}$ IS THEREFORE A RANDOM VARIABLE, FOR ANY $c \in C$. → SEE * BELOW

p_{sc} IS THE PRICE ONE MUST PAY TODAY FOR A UNIT CLAIM ON sc (i.e., TO OBTAIN $x_{sc} = 1$), AND THE PRICE RECEIVED TODAY FOR A PROMISE TO DELIVER ONE UNIT "TOMORROW."

NOT = RANDOM

ALL TRANSACTIONS (AND DOLLAR PAYMENTS) TAKE PLACE TODAY. ALL DELIVERIES x_{sc} TAKE PLACE TOMORROW IF AND ONLY IF THE DESIGNATED STATE s OCCURS. THE QUANTITIES (DELIVERIES) x_{sc} ARE CONTINGENT; THE PRICES p_{sc} (AND PAYMENTS) ARE NOT CONTINGENT.

* FOR A GIVEN $c \in C$, $(x_{sc})_{s\in S}$ IS A RANDOM VARIABLE:

$x_{sc} \in \mathbb{R}_+$ AND DEPENDS ON s.

$(x_{sc})_{(s,c)\in S\times C}$ IS A RANDOM VECTOR: THE VECTOR $(x_{sc})_{c\in C} \in \mathbb{R}_+^C$ AND DEPENDS ON s.

The <u>contingent claims model</u> is simply a reinterpretation of our standard Walrasian model:

Each consumer's UMP is the same as before:

$$\max u(x) \text{ s.t. } p_0 \cdot x_0 + \sum_{s \in S} \sum_{c \in C} p_{sc} x_{sc} \leq p_0 \cdot \mathring{x}_0 + \sum_{s \in S} \sum_{c \in C} p_{sc} \mathring{x}_{sc}.$$

Note that there is still just a <u>single</u> constraint.

An equilibrium is the same as before:

A pair $(p, x) \in \left(\mathbb{R}^C_+ \times \mathbb{R}^{S \times C}_+\right) \times \left(\mathbb{R}^{nC}_+ \times \mathbb{R}^{n(S \times C)}_+\right)$

that satisfies the (U-max) and (M-clr) conditions. The conditions are unchanged.

Note:

$$p = (p_0, (p_s)_{s \in S}) \in \mathbb{R}^C_+ \times \mathbb{R}^{S \times C}_+,$$

$$\text{where } p_0 = (p_{0c})_{c \in C} \in \mathbb{R}^C_+$$

$$p_s = (p_{sc})_{c \in C} \in \mathbb{R}^C_+, \ \forall s \in S$$

$$x^i = ((x^i_0, (x^i_s)_{s \in S}))_{i \in N} \in \mathbb{R}^{nC}_+ \times \mathbb{R}^{n(S \times C)}_+$$

$$\text{where } x^i_0 = (x^i_{0c})_{c \in C} \in \mathbb{R}^C_+$$

$$x^i_s = (x^i_{sc})_{c \in C} \in \mathbb{R}^C_+, \ \forall s \in S.$$

Only the notation is changed, from $k = 1, \ldots, \ell$ to $c \in C$ at $t=0$ and $sc \in S \times C$ at $t=1$.

THE PRICE OF AN EVENT-CONTINGENT CLAIM:

... IS JUST THE SUM OF THE A-D PRICES OF THE CORRESPONDING STATE-CONTINGENT CLAIMS:

$$P_E = \sum_{s \in E} P_s \quad \text{FOR AN EVENT } E \subseteq S$$

$$\left(\text{i.e., } P_{Ec} = \sum_{s \in E} P_{sc} \text{ FOR A GOOD } c \in C\right).$$

FOR EXAMPLE: (W/ ONLY ONE GOOD)

$S = \{H, M, L\}$; LET $E = \{H, M\}$ AND $E' = S = \{H, M, L\}$.

THEN $P_E = P_H + P_M$ AND $P_{E'} = P_H + P_M + P_L$.

THE PRICE OF A CONTRACT FOR DELIVERY IF $s \in E = \{H, M\}$

THE PRICE OF A CONTRACT FOR CERTAIN DELIVERY, i.e., IF $s \in E' = S$.

i.e., THESE ARE THE PRICES FOR DELIVERY OF EACH UNIT IN THE RESPECTIVE EVENTS.

BUT ARE THERE REALLY MARKETS LIKE THIS — i.e., A MARKET (FOR EACH $s \in S$ AND EACH $c \in C$) IN WHICH ONE CAN BUY OR SELL A CONTRACT FOR DELIVERY OF x_{sc} UNITS OF GOOD c IF AND ONLY IF STATE s OCCURS? AT A PRICE p_{sc} TO BE PAID TODAY? GENERALLY THERE AREN'T.

THE ARROW-DEBREU MODEL ASSUMES THERE <u>ARE</u> — NOT AS A MODEL OF REALITY, BUT AS A BENCHMARK AGAINST WHICH TO EVALUATE ALTERNATIVE MARKET STRUCTURES, AND AS A CONCEPTUAL DEVICE TO HELP US UNDERSTAND AND MODEL MARKETS (FOR COMMODITIES <u>AND</u> FOR SECURITIES) WHEN THERE IS UNCERTAINTY.