

CREDIT MARKET ONLY

THE CONSUMER CAN BORROW OR LEND - E.g.,
SELL OR BUY A BOND. THE RETURN NEXT PERIOD,
OR DIVIDEND, ON EACH UNIT IS $1+r$ IN EACH

STATE:

$$d = \begin{cases} [1+r] \leftarrow \text{IF } S=H \\ [1+r] \leftarrow \text{IF } S=L \end{cases}$$

LET y DENOTE UNITS PURCHASED TODAY, AT PRICE q .

INDIVIDUAL'S MARKET BEHAVIOR:

$$\begin{array}{ll} \max_{y, x_0, x_H, x_L} u(x_0, x_H, x_L) \text{ s.t.} & x_0 + qy \leq x_0^0 & \lambda_0 \\ & x_H \leq x_H^0 + (1+r)y & \lambda_H \\ & x_L \leq x_L^0 + (1+r)y & \lambda_L \end{array}$$

FOMC: (INTERIOR)

$$y: 0 = q\lambda_0 - (1+r)(\lambda_H + \lambda_L) \quad \text{i.e., } \frac{q}{1+r} = \frac{\lambda_H + \lambda_L}{\lambda_0}$$

$$x_0: u_0 = \lambda_0$$

$$x_H: u_H = \lambda_H$$

$$x_L: u_L = \lambda_L$$

COMBINING THE FOMC:

$$\frac{u_H + u_L}{u_0} = \frac{q}{1+r}$$

$$\text{i.e., } MRS_H + MRS_L = \frac{q}{1+r}$$

$$\text{IF } q=1: MRS_H + MRS_L = \frac{1}{1+r}.$$

↑ EACH UNIT COSTS \$1; y IS AMOUNT SAVED TODAY.

HENCEFORWARD LET $q=1$.

IF (y, x_0, x_H, x_L) SATISFIES ALL THREE CONSTRAINTS AS EQUATIONS, THEN WE HAVE

$$\begin{aligned}
 y &= \overset{\circ}{x}_0 - x_0 \quad (\text{SAVING}) \\
 x_H &= \overset{\circ}{x}_H + (1+r)(\overset{\circ}{x}_0 - x_0) \\
 x_L &= \overset{\circ}{x}_L + (1+r)(\overset{\circ}{x}_0 - x_0)
 \end{aligned}
 \quad \begin{aligned}
 x_S &= \overset{\circ}{x}_S + (1+r)y, \quad \forall S \\
 &= 15 + (1+r)(15 - x_0) \\
 &= 15 + (1+r)(15 - x_0).
 \end{aligned}$$

↑ IN OUR EXAMPLE

THE HAND EQUATIONS CAN BE WRITTEN

$$x_0 + \frac{1}{1+r} x_H = \overset{\circ}{x}_0 + \frac{1}{1+r} \overset{\circ}{x}_H$$

$$x_0 + \frac{1}{1+r} x_L = \overset{\circ}{x}_0 + \frac{1}{1+r} \overset{\circ}{x}_L$$

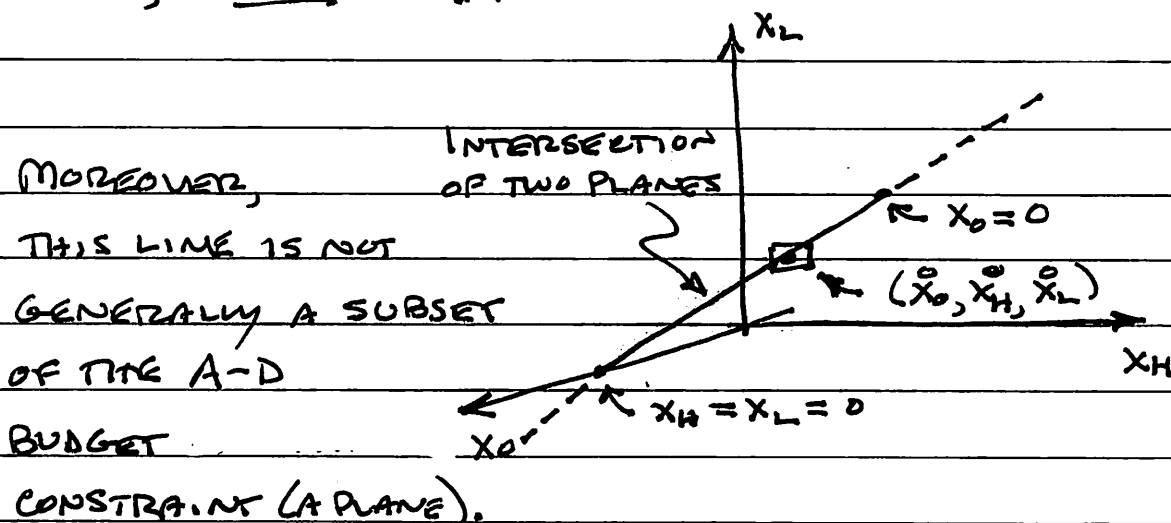
SO WE CAN WRITE THE MAXIMIZATION PROBLEM AS

$$\begin{aligned}
 \max_{x_0, x_H, x_L} u(x_0, x_H, x_L) \quad \text{s.t.} \quad & x_0 + \frac{1}{1+r} x_H = \overset{\circ}{x}_0 + \frac{1}{1+r} \overset{\circ}{x}_H \\
 & \text{AND } x_0 + \frac{1}{1+r} x_L = \overset{\circ}{x}_0 + \frac{1}{1+r} \overset{\circ}{x}_L
 \end{aligned}$$

↑ \mathbb{R}^3

TWO CONSTRAINTS (TWO PLANES IN \mathbb{R}^3).

⊛ THE BUDGET CONSTRAINT IS NOT A PLANE IN \mathbb{R}^3 BUT THE INTERSECTION OF TWO PLANES IN \mathbb{R}^3 — i.e., A LINE IN \mathbb{R}^3 .



ANOTHER EXAMPLE OF TWO

INTERSECTING CREDIT-MARKET HYPERPLANES

$\bar{x} = (\bar{x}_0, \bar{x}_H, \bar{x}_L) = (4, 4, 8)$ $r=1, q = \frac{1}{1+r} = \frac{1}{2}$ NOTE THAT $\bar{x}_H \neq \bar{x}_L$.
 $\underline{\quad}$

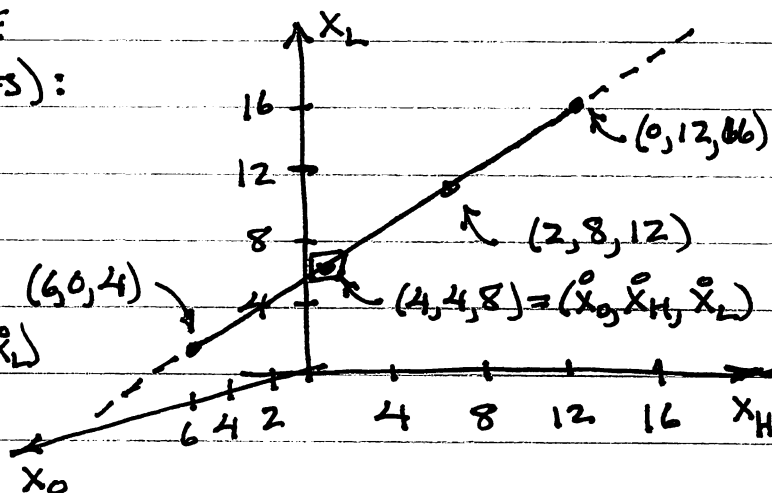
$x_0 + \frac{1}{2}x_H = \bar{x}_0 + \frac{1}{2}\bar{x}_H = 6$ $a^H = (1, q, 0) = (1, \frac{1}{2}, 0)$ $a^H \cdot x = 6$

$x_0 + \frac{1}{2}x_L = \bar{x}_0 + \frac{1}{2}\bar{x}_L = 8$ $a^L = (1, 0, q) = (1, 0, \frac{1}{2})$ $a^L \cdot x = 8$

NOTE THAT IF (x_0, x_H, x_L) IS ON BOTH CONSTRAINTS, THEN $x_L = x_H + 4$.

FOUR POINTS ON THE LINE (INTERSECTION OF PLANES):

x_0	x_H	x_L
0	12	16
2	8	12
4	4	8
6	0	4



THE SAME LINE IS OF COURSE THE INTERSECTION OF MANY (AN INFINITY!) OF OTHER PAIRS OF HYPERPLANES — FOR EXAMPLE,

$x_0 + \frac{1}{4}x_H + \frac{1}{4}x_L = 7$ AND $x_L = x_H + 4$

OR $x_0 + \frac{1}{2}x_H = 6$ AND $x_L = x_H + 4$

OR $x_0 + \frac{1}{2}x_L = 8$ AND $x_L = x_H + 4$,

ETC.

MARKET EQUILIBRIUM: (w/ CREDIT MARKET ONLY!)

$$MRS_H^i + MRS_L^i = \frac{1}{1+r}, \quad i = A, B.$$

$$\therefore MRS_H^A + MRS_L^A = MRS_H^B + MRS_L^B \quad (*)$$

NOTE: ALTHOUGH (*) IS IMPLIED BY THE PARETO CONDITION $MRS_S^A = MRS_S^B$ ($S = H, L$), IT DOES NOT IMPLY THE PARETO CONDITION — IT CAN HOLD WITHOUT THE PARETO CONDITION BEING SATISFIED.

\therefore THE CREDIT MARKET (ONLY) EQUILIBRIUM WILL NOT GENERALLY BE PARETO EFFICIENT.

IN OUR EXAMPLE:

$$(2 - \frac{1}{10} x_H^A) + (1 - \frac{1}{20} x_L^A) = (1 - \frac{1}{15} x_H^B) + (1 - \frac{1}{30} x_L^B) \quad (1)$$

$$\text{WE HAVE } x_H^A = x_H^0 + (1+r)(x_B^0 - x_B^A) = 15 + (1+r)(15 - x_0^A);$$

$$\text{ALSO } x_L^A = 15 + (15 - x_0^A), \text{ BECAUSE } x_H^0 = x_L^0 = 15 \text{ IN THIS EXAMPLE.}$$

LET'S DENOTE x_H^A AND x_L^A , WHICH ARE THE SAME, BY x_1^A .

SIMILARLY, $x_H^B = x_L^B = 15$; LET'S DENOTE x_H^B AND x_L^B , WHICH WILL HAVE TO BE THE SAME, BY x_1^B .

NOW EQUATION (1) IS

$$3 - \frac{3}{20} x_1^A = 2 - \frac{3}{30} x_1^B.$$

$$\text{BUT WE ALSO HAVE } x_1^A + x_1^B = 30; \text{ I.E., } x_1^B = 30 - x_1^A.$$

SO NOW EQUATION (1) BECOMES

$$\begin{aligned} 3 - \frac{3}{20}x_1^A &= 2 - \frac{3}{30}(30 - x_1^A) \\ &= 2 - 3 + \frac{1}{10}x_1^A = -1 + \frac{1}{10}x_1^A. \end{aligned}$$

$$\text{i.e., } \frac{5}{20}x_1^A = 4 \text{ ; i.e., } x_1^A = 16, x_1^B = 14.$$

THUS, WE HAVE $x_H^A = x_L^A = 16$ AND $x_H^B = x_L^B = 14$.

THESE ARE NOT THE CONSUMPTION LEVELS AT THE PARETO ALLOCATIONS WE OBTAINED EARLIER.

THUS,

→ THE CREDIT MARKET ALONE DOES NOT YIELD (IN OUR EXAMPLE) A PARETO EFFICIENT OUTCOME.

WE CAN ALSO SEE THE PARETO EFFICIENCY FAILURE BY CHECKING THE MRS VALUES:

$$MRS_H^A = 2 - \frac{16}{10} = \frac{2}{5} \quad MRS_L^A = 1 - \frac{16}{20} = \frac{1}{5}$$

$$MRS_H^B = 1 - \frac{14}{15} = \frac{1}{15} \quad MRS_L^B = 1 - \frac{14}{30} = \frac{8}{15}$$

WHICH ARE NOT THE MRS VALUES AT THE PARETO ALLOCATIONS.

WHAT IS THE EQUILIBRIUM INTEREST RATE?

$$\frac{1}{1+r} = MRS_H^i + MRS_L^i = \frac{3}{5}, \text{ so } 1+r = \frac{5}{3},$$

$$\text{AND } r = \frac{2}{3} = 66\frac{2}{3}\%.$$

HOW DOES THIS COMPARE WITH THE IMPLICIT INTEREST RATE AT THE PARETO ALLOCATIONS?

AT THE PARETO ALLOCATIONS WE OBTAINED

$$MRS_H^A = MRS_H^B = \frac{1}{5} \text{ AND } MRS_L^A + MRS_L^B = \frac{2}{5},$$

SO THE IMPLICIT INTEREST RATE SATISFIES

$$\frac{1}{1+r} = \frac{1}{5} + \frac{2}{5} = \frac{3}{5} \text{ AND } r = \frac{2}{3},$$

THE SAME AS THE EQUILIBRIUM INTEREST RATE

IN THE CREDIT MARKET. THAT IS JUST A

COINCIDENCE! IT'S NOT TYPICALLY TRUE THAT THE EQUILIBRIUM INTEREST RATE WITH (ONLY!) A CREDIT MARKET IS CONSISTENT WITH PARETO EFFICIENCY.

ALSO NOTE THAT WE DON'T GENERALLY HAVE $x_H^i = x_L^i$ AS WE DO HERE. WHAT WE HAVE, IN GENERAL, IS $x_S^i = x_S^0 + (1+r)y^i$ FOR EACH $S \in S$ — i.e.,

$$x_S^i = x_S^0 + (1+r)y^i;$$

THE INCREMENT TO x_S^0 PROVIDED BY PURCHASING y^i UNITS OF THE CREDIT INSTRUMENT IS THE SAME ACROSS STATES: WE OBTAINED $x_H^i = x_L^i$ BECAUSE $x_H^0 = x_L^0$.

FOR EXAMPLE, IF $x_H^0 = 25$ AND $x_L^0 = 10$, THEN

$$x_H^i = 25 + (1+r)y^i$$

$$x_L^i = 10 + (1+r)y^i.$$

TWO CREDIT INSTRUMENTS

w , q , R , AND $1+R$ ARE
IN THOUSANDS OF DOLLARS

BONDS:

PRICE IS FIXED AT \$1000; SAY $q \equiv 1$.

COUPONS R_t ($t=1, \dots, T$) WHICH PAY OUT R_t AT
TIME t .

PAYS BACK PRINCIPAL OF \$1000 AT TIME T ("MATURITY").

SIMPLIFY: ONLY COUPON IS R , PAID AT TIME T ,
SO THE ONLY RETURN IS $1+R$, AT TIME T .

IN THIS CASE WE ALSO SAY THAT R IS THE
INTEREST RATE.

T-BILLS:

PRICE IS NOT FIXED, DENOTE PRICE BY q .

PAYS OUT \$1000 AT TIME T , NO COUPONS.

SAY PAYOFF IS $1+R$ AT TIME T , WITH $R \geq 0$.

CONSUMER'S FOC:

$$\sum_{s \in S} MRS_s = \frac{q}{1+R}$$

WE SAY THAT THE INTEREST RATE (OR INTERNAL
RATE OF RETURN) IS $r = \frac{1+R}{q} - 1$.

NOTE THAT $1+r = \frac{1+R}{q}$, i.e., $\frac{q}{1+R} = \frac{1}{1+r}$.