EXAMPLE: EQUILIBRIUM AND EFFICIENCY WHEN THERE IS UNCERTAINTY

TWO CONSUMERS: $N = \{A, B\}$.
TWO PERIODS: $t=0$ (TODAY), $t=1$ (TOMORROW).
TWO POSSIBLE STATES: $s = H$ OR $s = L$ (SAY, TEMP $\leq 85°$ OR $> 85°$)
ONE ALL-PURPOSE GOOD (DOLLARS, FOR EXAMPLE).

CONSUMPTION PLAN FOR CONSUMER i: $x^i = (x_0^i, x_H^i, x_L^i) \in \mathbb{R}_+^3$.

PREFERENCES AND ENDOWMENTS:

$$\mathring{x}^A = (15, 15, 15) \quad \mathring{x}^B = (15, 15, 15) \quad \mathring{x} = \mathring{x}^A + \mathring{x}^B$$

$$\left.\begin{aligned} u^A(x) &= x_0 + 2x_H - \tfrac{1}{20}x_H^2 + x_L - \tfrac{1}{40}x_L^2 \\ u^B(x) &= x_0 + x_H - \tfrac{1}{30}x_H^2 + x_L - \tfrac{1}{60}x_L^2 \end{aligned}\right\}$$

SEE ASIDE ON NEXT PAGE FOR NOTE ON V-M UTILITIES

$$\therefore MRS_H^A = 2 - \tfrac{1}{10}x_H^A \qquad MRS_L^A = 1 - \tfrac{1}{20}x_L^A$$

$$MRS_H^B = 1 - \tfrac{1}{15}x_H^B \qquad MRS_L^B = 1 - \tfrac{1}{30}x_L^B$$

PARETO EFFICIENCY:

$MRS_H^A = MRS_H^B$: $2 - \frac{1}{10}x_H^A = 1 - \frac{1}{15}x_H^B$

SINCE $x_H^B = 30 - x_H^A$: $2 - \frac{1}{10}x_H^A = 1 - \frac{1}{15}(30 - x_H^A) = -1 + \frac{1}{15}x_H^A$

i.e., $\frac{5}{30}x_H^A = 3$; i.e., $\boxed{x_H^A = \frac{90}{5} = 18, \quad x_H^B = 12.}$

$MRS_L^A = MRS_L^B$: $1 - \frac{1}{20}x_L^A = 1 - \frac{1}{30}x_{BL}$; $\therefore \frac{1}{20}x_L^A = \frac{1}{30}x_L^B$

i.e., $x_L^A = \frac{2}{3}x_L^B$; $\therefore \boxed{x_L^A = 12, \quad x_L^B = 18.}$

x_0^A AND x_0^B ONLY HAVE TO SATISFY $x_0^A + x_0^B = \mathring{x}_0^A + \mathring{x}_0^B = 30$.

EFFICIENCY PRICES:

$MRS_H^A = MRS_H^B = \frac{1}{5} \qquad MRS_L^A = MRS_L^B = \frac{2}{5}$.

WALRASIAN EQUILIBRIUM:

THE UTILITY FUNCTIONS ARE LOCALLY NON-SATIATED, SO THE FIRST WELFARE THEOREM TELLS US THAT AN EQUILIBRIUM WILL BE PARETO EFFICIENT. THEREFORE WE HAVE $(x_H^A, x_L^A) = (18, 12)$ AND $(x_H^B, x_L^B) = (12, 18)$. MOREOVER, EQUILIBRIUM PRICES WILL BE (PROPORTIONAL TO) THE EFFICIENCY PRICES: $(p_H, p_L) = (\frac{1}{5}, \frac{2}{5})$ IF $p_0 = 1$.

FINALLY, THE CONSUMPTIONS AT $t=0$ ARE

$$x_0^A = \mathring{x}_0^A - p_H(x_H^A - \mathring{x}_H^A) - p_L(x_L^A - \mathring{x}_L^A)$$

$$= 15 - \frac{1}{5}(3) - \frac{2}{5}(-3) = 15 - \frac{3}{5} + \frac{6}{5} = 15 + \frac{3}{5} = 15\tfrac{3}{5}.$$

$$x_0^B = 15 - \frac{1}{5}(-3) - \frac{2}{5}(3) = 15 + \frac{3}{5} - \frac{6}{5} = 15 - \frac{3}{5} = 14\tfrac{2}{5}.$$

ASIDE: (VON NEUMANN-MORGENSTERN UTILITIES)

CONSUMER A'S PREFERENCE CAN BE REPRESENTED BY A VON NEUMANN-MORGENSTERN (V-M) UTILITY FUNCTION; i.e., THERE IS A V-M UTILITY FUNCTION $v: \mathbb{R} \to \mathbb{R}$ AND PROBABILITIES π_H AND π_L SUCH THAT

$$u^A(x_0, x_H, x_L) = x_0 + \pi_H v(x_H) + \pi_L v(x_L)$$

— NAMELY, $v(z) = 3z - \frac{3}{40}z^2$ AND $\pi_H = \frac{2}{3}$, $\pi_L = \frac{1}{3}$.

BUT CONSUMER B'S PREFERENCE IS NOT REPRESENTABLE BY A V-M UTILITY FUNCTION: THERE DO NOT EXIST $v(\cdot)$, π_H, AND π_L SUCH THAT

$$u^B(x_0, x_H, x_L) = x_0 + \pi_H v(x_H) + \pi_L v(x_L).$$

HOWEVER, THIS DOES NOT MEAN THAT u^B IS STRANGE OR PATHOLOGICAL; IT SIMPLY ISN'T OF THE V-M FORM.

THE CONSUMER'S DECISION PROBLEM:

$$\max_{x_0, x_H, x_L} u(x) \text{ s.t. } x_0 + p_H x_H + p_L x_L = \mathring{x}_0 + p_H \mathring{x}_H + p_L \mathring{x}_L$$

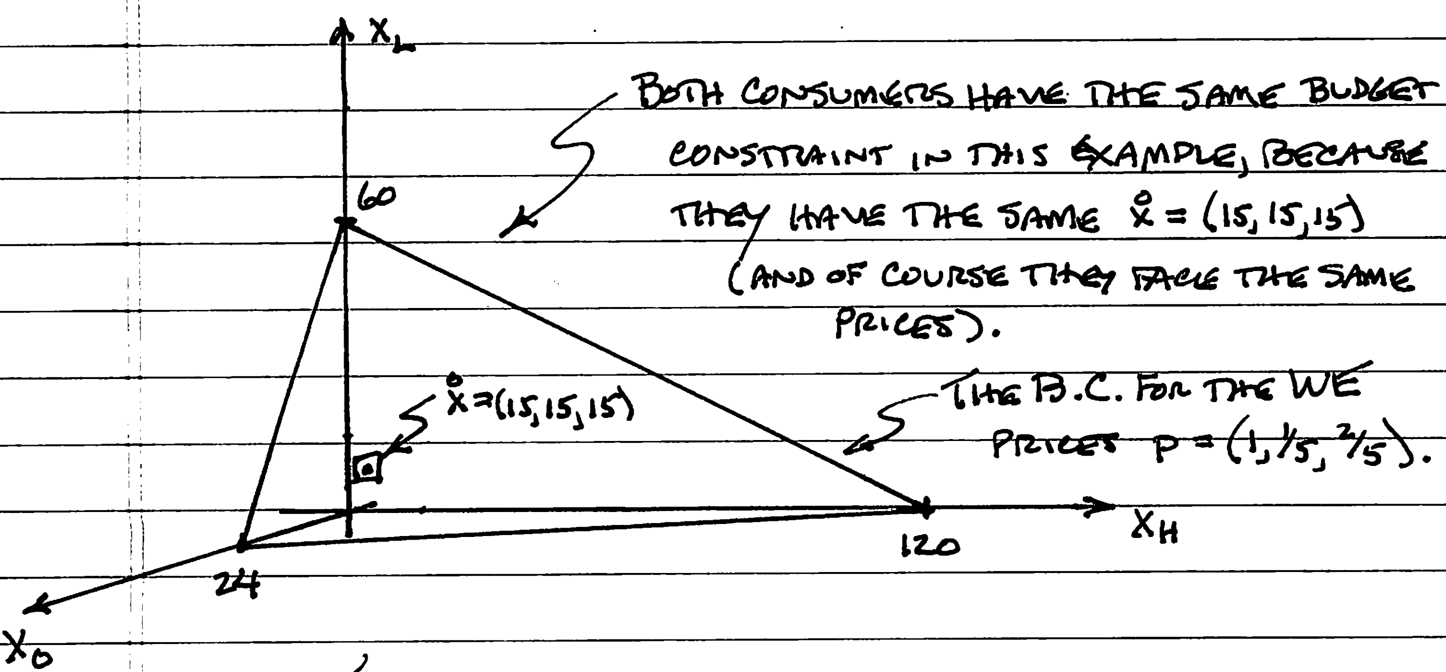
$$\text{i.e., } \quad x_0 + \tfrac{1}{5} x_H + \tfrac{2}{5} x_L = \mathring{x}_0 + \tfrac{1}{5} \mathring{x}_H + \tfrac{2}{5} \mathring{x}_L$$

$$= 15 + 3 + 6 = 24.$$

(AT THE WE PRICES 1, 1/5, 2/5.)

THE CONSUMER'S BUDGET CONSTRAINT:

THE AXIS INTERCEPTS: $x_0 = 24$, $x_H = (5)(24) = 120$, $x_L = (\frac{5}{2})(24) = 60$.



THIS GEOMETRY IS GOING TO BE VERY USEFUL IN A LITTLE WHILE!

FOR DIFFERENT PRICE LISTS $p = (p_0, p_H, p_L)$ THE BUDGET CONSTRAINT (THE TRIANGLE) WILL MOVE, BUT IT WILL ALWAYS CONTAIN $\mathring{x} = (\mathring{x}_0, \mathring{x}_H, \mathring{x}_L)$ — i.e., (15, 15, 15) IN OUR EXAMPLE.

MARGINAL RATE OF SUBSITITUTION AND PRESENT VALUE

OUR CONVENTION SO FAR HAS BEEN TO EXPRESS VALUES IN TERMS OF UNITS OF THE Y-GOOD:

$$MRS := \frac{u_x}{u_y} \qquad \text{FOMC: } MRS = \frac{P_x}{P_y}$$

$= -\frac{\partial y}{\partial x}$, VALUE OF A UNIT OF X-GOOD AS # UNITS OF Y-GOOD YOU'RE WILLING TO GET/GIVE UP

MARKET VALUE OF X-GOOD: # UNITS OF Y-GOOD THAT TRADE FOR A UNIT OF X-GOOD.

INTERTEMPORAL:

$$MRS = \frac{u_0}{u_1} \qquad \text{FOMC: } MRS = 1+r$$

THIS MEASURES VALUE ~~IN~~ IN TERMS OF TOMORROW'S VALUE.

BUT WE GENERALLY WANT TO USE PRESENT VALUE, i.e, MEASURE VALUES IN TERMS OF TODAY'S GOOD:

$$MRS_{10} := \frac{u_1}{u_0} \qquad \text{FOMC: } MRS_{10} = \frac{1}{1+r}$$

LIKE $\frac{P_1}{P_0}$ OR $\frac{P_y}{P_x}$

WHEN THERE IS ONLY ONE GOOD (e.g., DOLLARS), BUT MULTIPLE POSSIBLE STATES TOMORROW, OR MULTIPLE FUTURE PERIODS, WE WILL OMIT THE "0" SUBSCRIPT ON THE MRS:

$$MRS_\theta,\ \theta \in \Theta \quad \text{OR} \quad MRS_t,\ t \in T.$$