MODELING UNCERTAINTY AND INFORMATION

$s \in S$: STATES OF THE WORLD (SAMPLE SPACE; ELEMENTARY EVENTS)

- ONE AND ONLY ONE WILL OCCUR.
- WE DON'T KNOW TODAY (WHEN MAKING DECISION) WHICH ONE IT WILL BE.

S CAN BE FINITE, INFINITE, A CONTINUUM.
(WE WILL CONSIDER ONLY FINITE SETS S)

A DECISION-MAKER'S "BELIEF" ABOUT S IS REPRESENTED BY A PROBABILITY MEASURE ON S.
(WE WILL NOT NEED TO EXPLICITLY CONSIDER PROBABILITIES.)

$(x_s)_{s \in S} \in \mathbb{R}_+^{CS} = (\mathbb{R}_+^{C})^{S}$: A PLAN/CONTINGENT PLAN/STRATEGY.

NOTE THAT $(x_s)_S$ IS JUST A FUNCTION FROM S TO $\mathbb{R}_+^C$ — WE COULD WRITE IT $x(s)$ INSTEAD OF x_s. THIS MAKES IT CLEAR THAT A PLAN $(x_s)_S$ IS A RANDOM VARIABLE (OR OFTEN CALLED A RANDOM VECTOR), $x : S \to \mathbb{R}_+^C$.

TEMPORAL RESOLUTION OF UNCERTAINTY

BECOMING INFORMED; OBTAINING INFORMATION

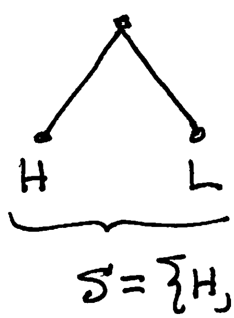
MODEL VIA

(1) A SET $\mathcal{T}=\{0,1,\ldots,T\}$ OF DATES, AND

(2) A TREE STRUCTURE OR PARTITION OF S.

GEOMETRIC ALGEBRAIC

EXAMPLE 1:



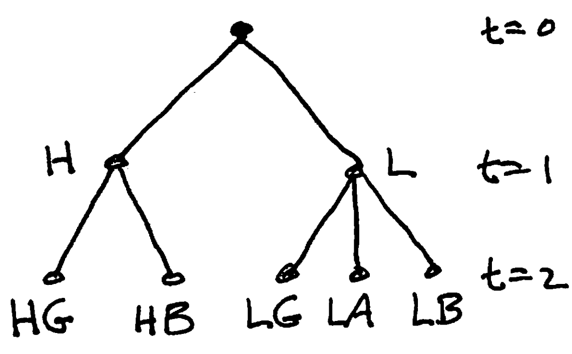
$S=\{H,L\}$, $\mathcal{T}=\{0,1\}$

PARTITIONS:

$\mathcal{S}_0=\{S\}$

$\mathcal{S}_1=\{\{H\},\{L\}\}$

EXAMPLE 2:



$S=\{HG,HB,LG,LA,LB\}$, $\mathcal{T}=\{0,1,2\}$

$\mathcal{S}_0=\{S\}$

"H" "L"

$\mathcal{S}_1=\{H,L\}=\{\{HG,HB\},\{LG,LA,LB\}\}$

$\mathcal{S}_2=\{\{HG\},\{HB\},\{LG\},\{LA\},\{LB\}\}$

$\mathcal{S}_0$ IS COARSEST

$\mathcal{S}_2$ IS FINEST

$t'>t \Rightarrow \mathcal{S}_{t'}$ IS A REFINEMENT OF $\mathcal{S}_t$, i.e.,

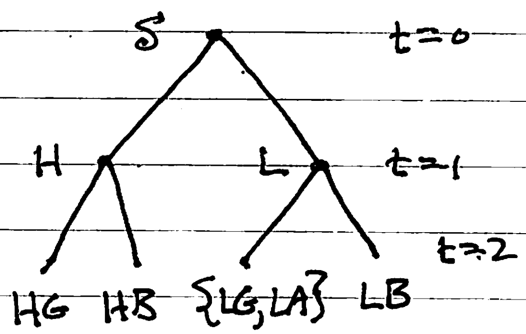
$$\forall E'\in\mathcal{S}_{t'}: \exists E\in\mathcal{S}_t: E'\subseteq E.$$

FOR EXAMPLE, WE CAN'T HAVE $\mathcal{S}_2$ INCLUDE ~~###~~ THE EVENT $\{HG,LG\}=E'$ IF HG AND LG ARE IN DIFFERENT EVENTS IN $\mathcal{S}_1$.

ASYMMETRIC INFORMATION:

THE TERM "ASYMMETRIC INFORMATION" MEANS PEOPLE HAVE DIFFERENT INFORMATION. FOR EXAMPLE, SUPPOSE ONE PERSON'S INFORMATION IS AS IN EXAMPLE 2 AND ANOTHER PERSON'S IS AS IN THE FOLLOWING EXAMPLE.

EXAMPLE 3:



$\mathcal{S}_0 = \{S\}$

$\mathcal{S}_1 = \{H, L\} = \{\{HG, HB\}, \{LG, LA, LB\}\}$

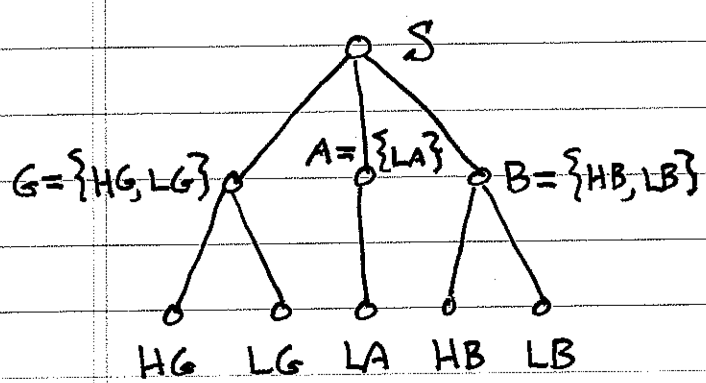
$\mathcal{S}_2 = \{\{HG\}, \{HB\}, \{LG, LA\}, \{LB\}\}$

WE WOULD SAY THAT THIS PERSON HAS LESS INFORMATION AT $t=2$ THAN THE PERSON IN EXAMPLE 2, BECAUSE THE PARTITION $\mathcal{S}_2$ IN EXAMPLE 2 (LET'S DENOTE IT BY $\mathcal{S}_2'$) IS STRICTLY FINER THAN THE $\mathcal{S}_2$ IN THIS EXAMPLE.

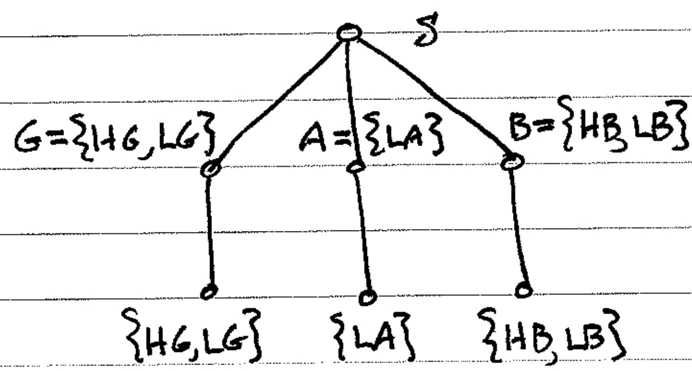
Differential ("Asymmetric") Information

In our Example 2, information was discovered or "revealed" in a particular order: First it was discovered whether H or L is true (i.e., whether $s \in H = \{HG, HB\}$ or $s \in \{LG, LA, LB\}$), and then whether G, A, or B is true (Thus, at $t=2$ it was known exactly which member of $S = \{HG, HB, LG, LA, LB\}$ is the true state s). But different individuals might, at any stage, have <u>different</u> information which is nevertheless <u>consistent</u> with others' information. Here are some examples of other resolutions of the uncertainty in Example 2. Notice that whichever member of S is the true state s, we can identify "what each individual will know" at each stage.

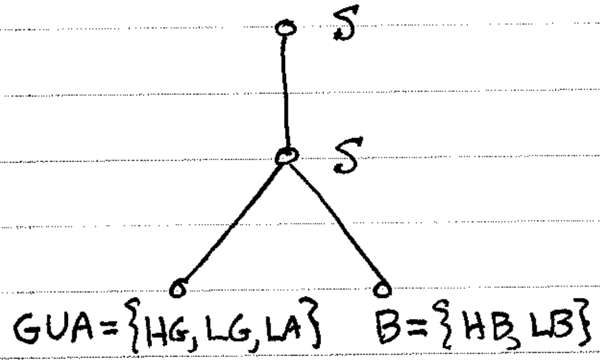
(a) "Observe whether G, A, or B; then whether H or L."



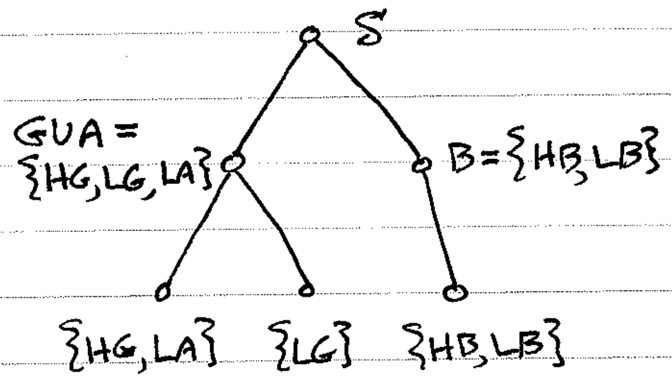
(b) "Observe <u>only</u> whether G, A, or B — at $t=1$."



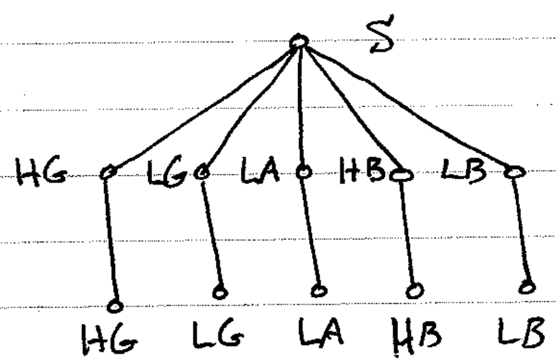
(c) "OBSERVE ONLY WHETHER B OR 'G OR A' — AT t=2."



(d) "OBSERVE WHETHER B OR 'G OR A'; THEN (IF 'G OR A') WHETHER 'L AND G' OR NOT."



(e) "OBSERVE THE WHOLE STATE AT t=1."



IN TERMS OF PARTITIONS, (d) FOR EXAMPLE WOULD BE

$$\mathcal{S}_0 = S$$
$$\mathcal{S}_1 = \{\{HG,LG,LA\},\{HB,LB\}\}$$
$$\mathcal{S}_2 = \{\{HG,LA\},\ \{LG\},\ \{HB,LB\}\}.$$