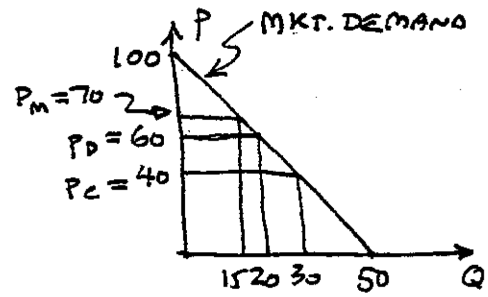
Duopoly Example

(Cournot Analysis)



The market situation:

$p = 100 - 2Q, \quad Q = q_1 + q_2$

$c_i(q_i) = 40q_i, \; i = 1,2.$

Each firm takes other's quantity choice as "given," unaffected by own choice.

Product is homogeneous.

Firm 1's problem:

$$p = 100 - 2(q_1 + q_2)$$
$$= [100 - 2q_2] - 2q_1$$

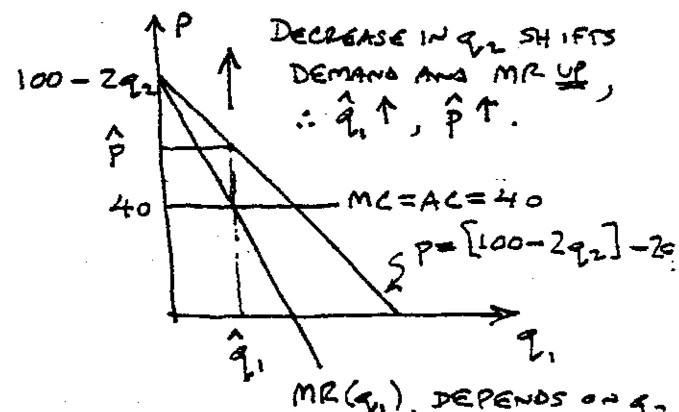
$$\therefore MR(q_1) = [100 - 2q_2] - 4q_1$$

$MR = MC$: $100 - 2q_2 - 4q_1 = 40$

i.e., $4q_1 + 2q_2 = 60$

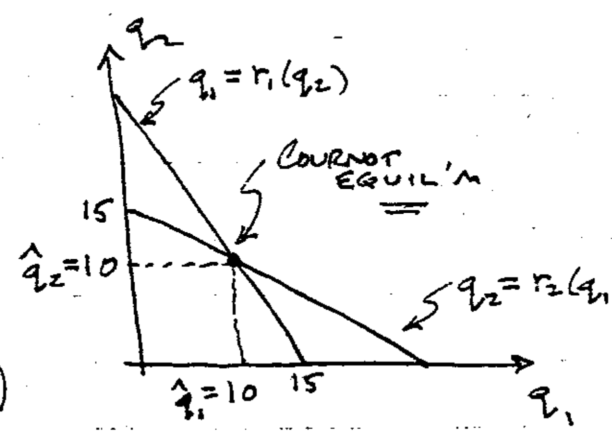
i.e., $\boxed{q_1 = 15 - \frac{1}{2}q_2}$

Firm 1's "reaction function" or "reaction curve."



Firm 2: $2q_1 + 4q_2 = 60$

i.e., $\boxed{q_2 = 15 - \frac{1}{2}q_1}$



(They're the same because firms' costs are the same — and because $p = 100 - 2(q_1 + q_2)$ [good is homogeneous].)

Can solve the two equations simultaneously, obtaining

$q_1 = 10, \; q_2 = 10. \quad \therefore Q = 20, \; p = 60.$

[Put back into individual firm's problem to check that this is right.]

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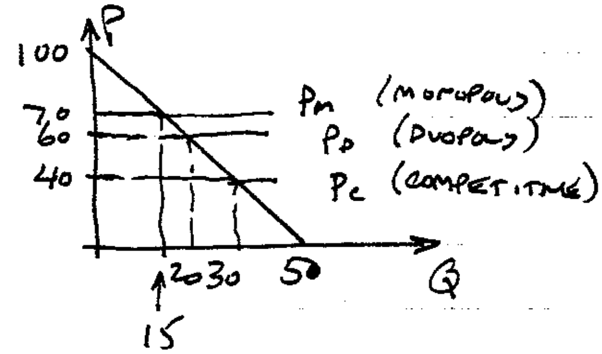


$$\pi_1(q_1, q_2) = pq_1 - C_1(q_1)$$
$$= [100 - 2(q_1+q_2)]q_1 - 40q_1$$
$$= 100q_1 - 2q_1^2 - 2q_2q_1 - 40q_1$$

$$\frac{\partial \pi_1}{\partial q_1} = 100 - 4q_1 - 2q_2 - 40 = 60 - 4q_1 - 2q_2$$
$$= 0 \iff 4q_1 = 60 - 2q_2; \text{ i.e., } q_1 = 15 - \tfrac{1}{2}q_2.$$

Similarly, $\frac{\partial \pi_2}{\partial q_2} = 0 \iff q_2 = 15 - \frac{1}{2}q_1.$

$\therefore$ Equilibrium is $q_1 = q_2 = 10, \quad Q = 20, \quad p = \$60;$

$\pi_1 = \pi_2 = pq_i - C(q_i) = (\$60)(10) - (\$40)(10) = \$200.$

If a monopoly (cartel, collusion):

$MR = 100 - 4Q; \quad MR(Q) = MC(Q): \; 100 - 4Q = 40;$

i.e., $60 = 4Q$; i.e., $\underline{Q = 15}$

$Q = 15, \; p = \$70, \; \pi = \$450.$

Note that $\$450 > \$200 + \$200.$

If competitive: $p = MC = \$40; \; Q = 30; \; \pi = 0.$

COURNOT EQUILIBRIUM: DEFINITION

DEFN: A PAIR OF QUANTITIES $(\hat{q}_1, \hat{q}_2)$ IS A COURNOT EQUILIBRIUM IF

$\hat{q}_1$ MAXIMIZES $\pi_1(q_1, \hat{q}_2)$

AND $\hat{q}_2$ MAXIMIZES $\pi_2(\hat{q}_1, q_2)$.

MORE GENERALLY (WITH n FIRMS):

DEFN: A LIST $(\hat{q}_1, \ldots, \hat{q}_n)$ OF QUANTITIES IS A COURNOT EQUILIBRIUM IF

$\forall i$: $\hat{q}_i$ MAXIMIZES $\pi_i(q_i, \hat{q}_{\sim i})$.

$(\hat{q}_1, \ldots, \hat{q}_{i-1}, \hat{q}_{i+1}, \ldots, \hat{q}_n)$

Monopoly in the Example

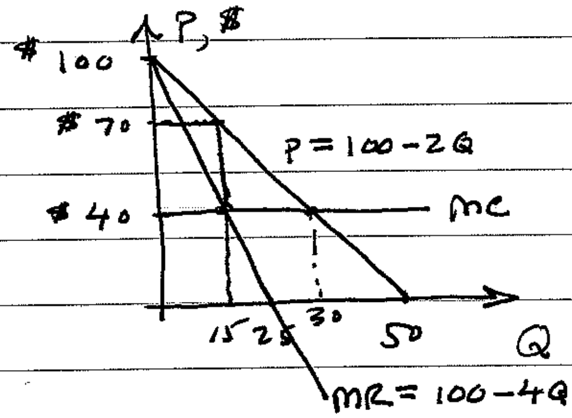
$P = 100 - 2Q$

$R(Q) = P \cdot Q = (100 - 2Q)Q = 100Q - 2Q^2$

$R'(Q) = 100 - 4Q = MR$

$C(Q) = 40Q$

$MC = 40$



$MR = MC: \; 100 - 4Q = 40$

$\therefore \; 4Q = 60$

$Q = 15, \; P = \$70$

$R = \$1050, \; C = \$600$

$\pi = \$450 = PS$

$CS = \frac{1}{2}(\$30)(15) = \$225$

TOTAL SURPLUS =

$= CS + PS$

$= \$225 + \$450$

$= \$675.$

Competitive Outcome

$P = MC = \$40, \; Q = 30$

$R = \$1200, \; C = \$1200$

$\pi = 0, \; PS = 0$

$CS = \frac{1}{2}(\$60)(30) = \$900$

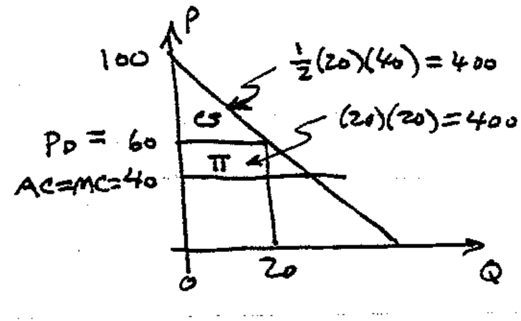
TOTAL SURPLUS =

$= CS + PS$

$= \$900 + 0 = \$900$

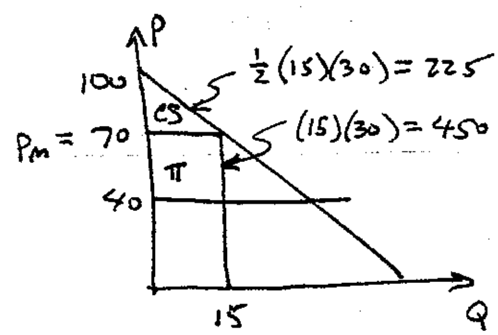
Duopoly:

Consumer surplus = 400
Producer surplus = 400
Total surplus = 800



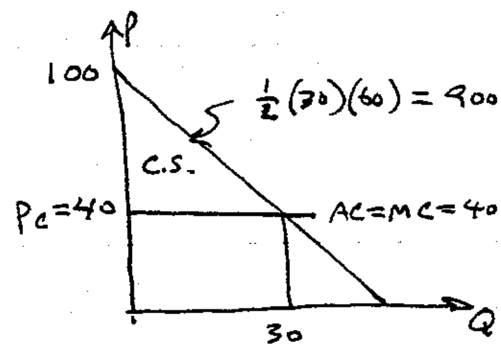
Monopoly:

Consumer surplus = 225
Producer surplus = 450
Total surplus = 675



Competitive:

Consumer surplus = 900
Producer surplus = 0
Total surplus = 900



SUPPOSE THIS MARKET DEMAND COMES FROM 5 IDENTICAL CONSUMERS:

MARKET DEMAND: $p = 100 - 2Q$; i.e., $Q = 50 - \frac{1}{2}p$.

EACH CONSUMER: $x = \frac{1}{5}Q = 10 - \frac{1}{10}p$

i.e., $p = 100 - 10x$. (SEE DEMAND CURVE, BELOW)

BUT HE CHOOSES WHERE $MRS = p$;

$\therefore$ $MRS = 100 - 10x$

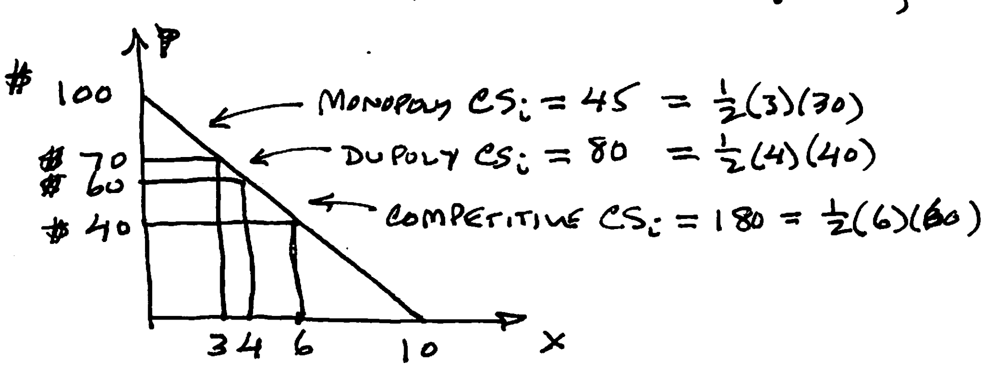
$\therefore u(x,y) = y + 100x - 5x^2$. LET $\mathring{y} = 1000$.

IF NO PURCHASE OF X: $x = 0$, $y = 1000$, $u = 1000$.

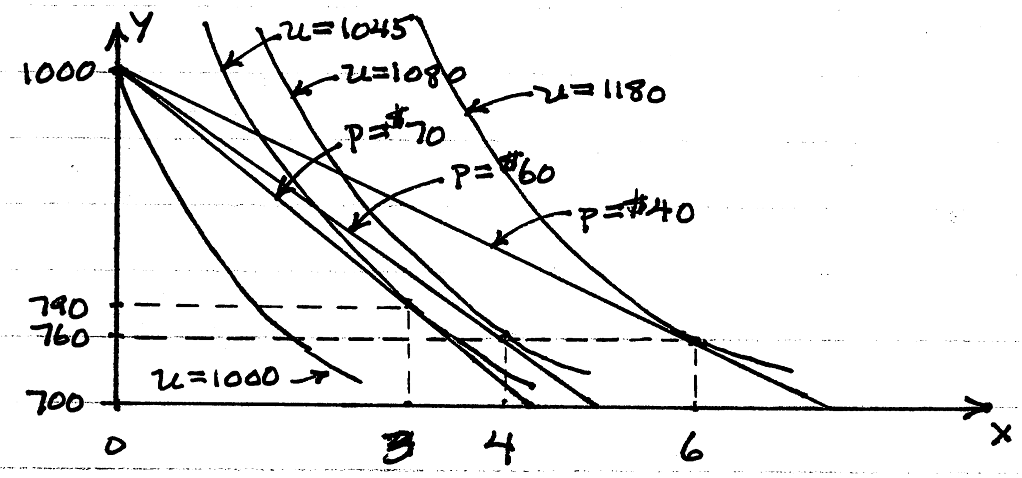
MONOPOLY: $p = \$70$, $\therefore x = 3$, $\therefore u = 1000 - 210 + 300 - 45 = 1045$
$Q = 15$ $\therefore CS_i = 45$ AND $CS = 225$.

DUOPOLY: $p = \$60$, $\therefore x = 4$, $\therefore u = 1000 - 240 + 400 - 80 = 1080$
$Q = 20$ $\therefore CS_i = 80$ AND $CS = 400$

COMPETITIVE: $p = \$40$, $\therefore x = 6$, $\therefore u = 1000 - 240 + 600 - 180 = 1180$
$Q = 30$ $\therefore CS_i = 180$, AND $CS = 900$

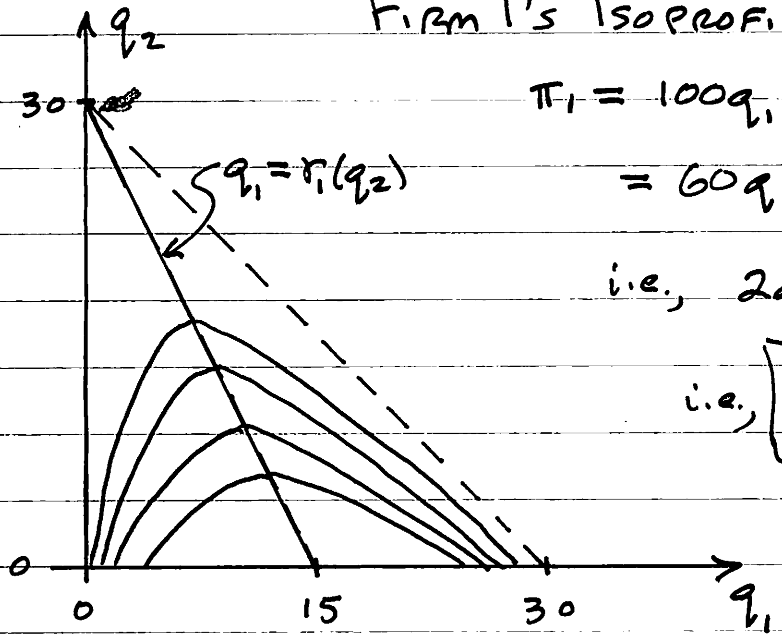


EACH CONSUMER:



Isoprofit Curves

Firm 1's Isoprofit Curves:



$$\pi_1 = 100q_1 - 2q_2q_1 - 2q_1^2 - 40q_1$$
$$= 60q_1 - 2q_2q_1 - 2q_1^2$$

i.e., $2q_2q_1 = 60q_1 - 2q_1^2 - \pi_1$

i.e., $\boxed{q_2 = 30 - q_1 - \frac{\pi_1}{2q_1}}$

For any given value of q_2, the best choice for q_1 (i.e., the one on the best isoprofit curve available) is $q_1 = r_1(q_2)$.

Putting the two firms' isoprofit maps together: