THE DUOPOLY OUTCOME ACCORDING TO THE COURNOT MODEL

SUPPOSE THERE ARE TWO FIRMS ON THE ISLAND, NUMERO UNO AND DOS HERMANOS, CAPABLE OF PRODUCING WINE, EACH AT $4 PER GALLON. WHAT WILL BE THE OUTCOME — i.e., THE PRICE, THE FIRMS' PRODUCTION LEVELS, AND THE WELFARE OF THE CONSUMERS AND THE FIRMS' OWNERS? THE COURNOT MODEL IS THE PLACE TO BEGIN TRYING TO ANALYZE THIS SITUATION: ALTHOUGH THE COURNOT MODEL LACKS REALISM IN CERTAIN IMPORTANT RESPECTS, IT IS THE FOUNDATION UPON WHICH ONE CAN DEVELOP AN ANALYSIS THAT <u>IS</u> MORE REALISTIC.

THE BASIC ASSUMPTIONS OF THE COURNOT MODEL ARE THAT

(1) EACH FIRM RECOGNIZES THAT IT FACES A DOWNWARD-SLOPING DEMAND CURVE — SPECIFICALLY, THAT THE MORE THE FIRM OFFERS FOR SALE, THE LOWER WILL BE THE RESULTING MARKET PRICE; AND

(2) NEITHER FIRM TAKES ACCOUNT OF THE LIKELIHOOD THAT ITS OWN CHOICE OF AN OUTPUT LEVEL WILL ELICIT A REACTION FROM ITS RIVAL FIRM (THIS IS THE UNREALISTIC PART).

LET'S DENOTE THE TWO FIRMS' OUTPUT CHOICES AS

Q_1, NUMEROUNO'S OUTPUT LEVEL,

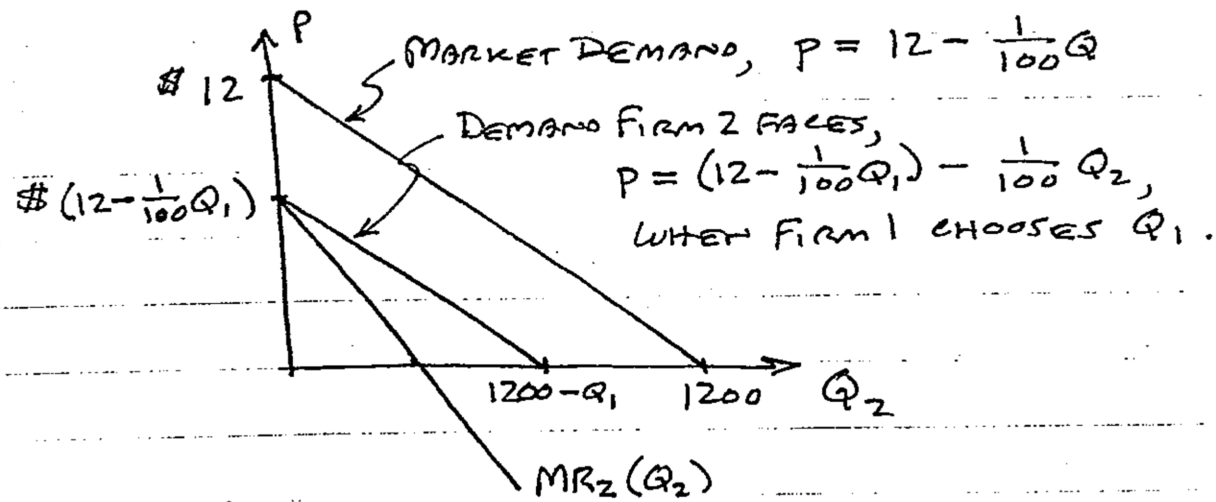
AND Q_2, DOS HERMANOS' OUTPUT LEVEL.

THUS, THE RESULTING AMOUNT SUPPLIED IN THE MARKET IS $Q = Q_1 + Q_2$.

THE MARKET DEMAND CURVE IS, AS BEFORE,

$$p = 12 - \frac{1}{100} Q$$
$$= 12 - \frac{1}{100}(Q_1 + Q_2)$$
$$= 12 - \frac{1}{100} Q_1 - \frac{1}{100} Q_2 .$$

FOCUSING OUR ATTENTION FIRST ON DOS HERMANOS'S DECISION PROBLEM, THE ASSUMPTIONS (1) AND (2) SAY THAT DOS HERMANOS TREATS Q_1 AS A "PARAMETER," OR A CONSTANT, UNAFFECTED BY ITS OWN CHOICE OF AN OUTPUT LEVEL Q_2:



NOW DOS HERMANOS'S DECISION PROBLEM IS FAMILIAR: IT SIMPLY WANTS TO CHOOSE THE Q_2-LEVEL THAT MAXIMIZES ITS PROFIT, i.e., AT WHICH $MR_2 = MC$:

$$MR_2(Q_2) = MC$$

$$\text{i.e.,}\quad \left(12 - \frac{1}{100}Q_1\right) - \frac{2}{100}Q_2 = 4$$

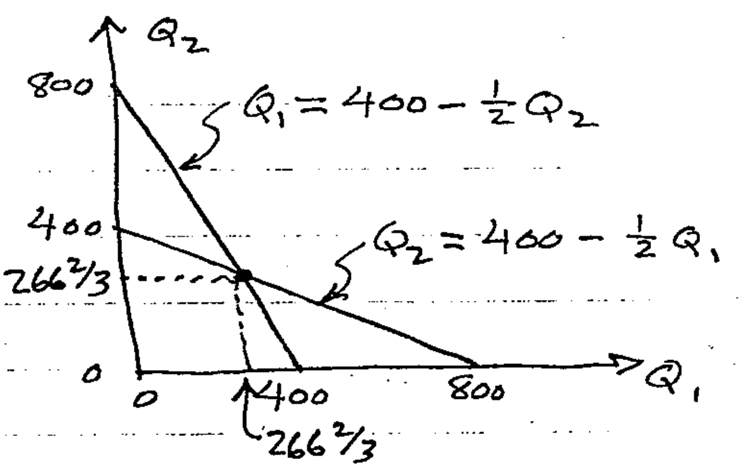
$$\text{i.e.,}\quad \frac{2}{100}Q_2 = 8 - \frac{1}{100}Q_1$$

$$\text{i.e.,}\quad \boxed{Q_2 = 400 - \tfrac{1}{2}Q_1.}$$

WE REFER TO THIS AS FIRM 2'S <u>REACTION FUNCTION</u>, SHOWING FIRM 2'S CHOICE AS A FUNCTION OF WHAT FIRM 1 IS CHOOSING. SIMILARLY, FIRM 1'S REACTION FUNCTION IS

$$\boxed{Q_1 = 400 - \tfrac{1}{2}Q_2.}$$

WE CAN DEPICT THESE TWO REACTION FUNCTIONS IN A DIAGRAM:



.... AND IT IS EASY TO SEE THAT THE ONLY "SOLUTION" OR "EQUILIBRIUM" IS $Q_1 = Q_2 = 266\frac{2}{3}$.

$\therefore Q = 533\frac{1}{3}$, $p = \$6\frac{2}{3}$.

EACH FIRM'S PROFIT IS

$$\pi_i = (p - AC) \cdot Q_i$$
$$= (\$6\tfrac{2}{3} - \$4)(266\tfrac{2}{3})$$
$$= (\$2\tfrac{2}{3})(266\tfrac{2}{3}) = \$711\tfrac{1}{9}.$$

PRODUCER SURPLUS (RENTS TO PRODUCERS) IS THEREFORE

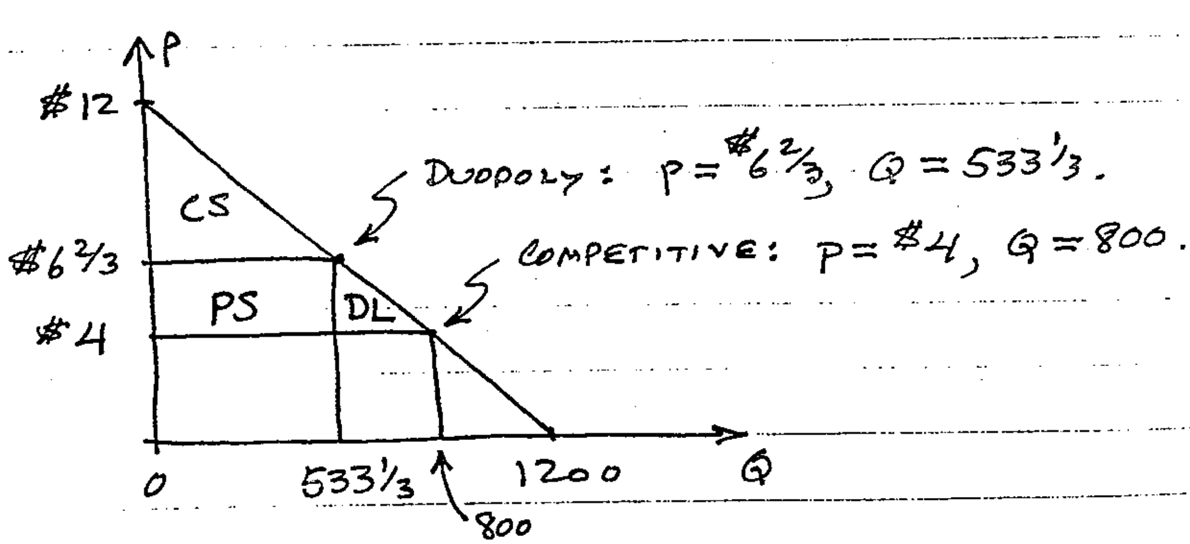
$$PS = \$711\tfrac{1}{9} + \$711\tfrac{1}{9} = \$1422\tfrac{2}{9}.$$

CONSUMER SURPLUS IS

$$CS = \tfrac{1}{2}(\$12 - \$6\tfrac{2}{3})(533\tfrac{1}{3})$$
$$= \$1422\tfrac{2}{9}.$$

DEADWEIGHT LOSS IS

$$DL = \$3200 - (CS + PS)$$
$$= \$3200 - \$2844\tfrac{4}{9} = \$355\tfrac{5}{9}.$$



YOU CAN DO EXACTLY THE SAME KIND OF ANALYSIS FOR THREE FIRMS, FOR FOUR FIRMS, ..., AND FOR n FIRMS (i.e., AN ARBITRARY NUMBER OF FIRMS). FIRM 1, FOR EXAMPLE, FACES THE DEMAND CURVE

$$p = \left(12 - \frac{1}{100}\sum_{i=2}^{n} Q_i\right) - \frac{1}{100} Q_1 ,$$

AND THE REST OF THE ANALYSIS WORKS THE SAME WAY AS WE'VE ALREADY DONE IT FOR THE CASE $n=2$.

THE SOLUTION YOU GET IS

$$Q_i = \frac{800}{n+1} \text{ FOR EACH FIRM } i; \quad \pi_i = \frac{2}{(n+1)^2}\cdot 3200;$$

$$Q = \left(\frac{n}{n+1}\right)\cdot 800, \qquad p = 12 - \left(\frac{n}{n+1}\right)\cdot 8$$

$$CS = \frac{n^2}{(n+1)^2}\cdot \#3200$$

$$PS = \frac{2n}{(n+1)^2}\cdot \#3200$$

$$DL = \frac{1}{(n+1)^2}\cdot \#3200$$

NOTICE THAT, AS n (THE NUMBER OF FIRMS) GROWS LARGE, CS GETS NEAR #3200, PS GETS NEAR 0, AND DL GETS NEAR 0 VERY QUICKLY (e.g., IF $n=7$, THEN DL = #50).