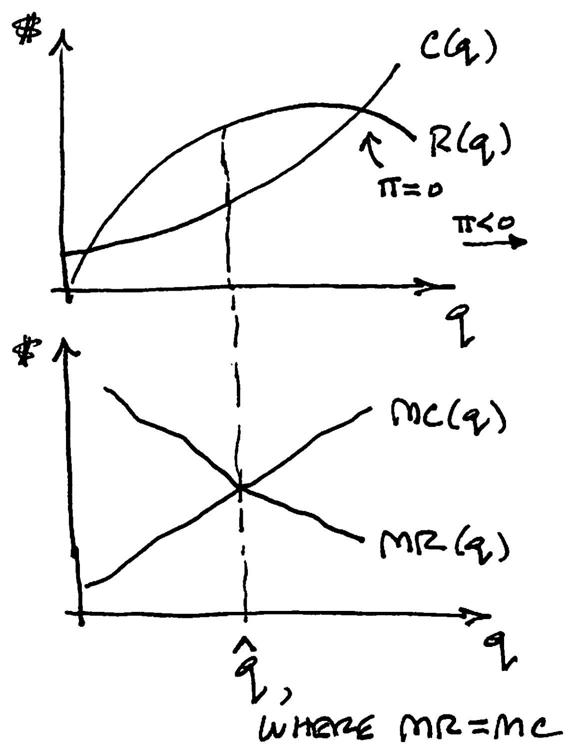
THE ELEMENTARY THEORY OF THE FIRM

IN THE MOST BASIC THEORY OF THE FIRM, THE FIRM PRODUCES ONE PRODUCT AND CHOOSES ITS LEVEL OF OUTPUT q, TO MAXIMIZE ITS PROFIT, $\pi(q)$:



$$\max_q \pi(q) := R(q) - C(q)$$

FOMC: $\pi'(q) \leq 0$ & $\pi'(q) = 0$ IF $q > 0$

i.e., $R'(q) - C'(q) = 0$

i.e., $MR(q) = MC(q)$

SECOND-ORDER CONDITION:

$\pi''(q) < 0$

i.e., $MR'(q) < MC'(q)$

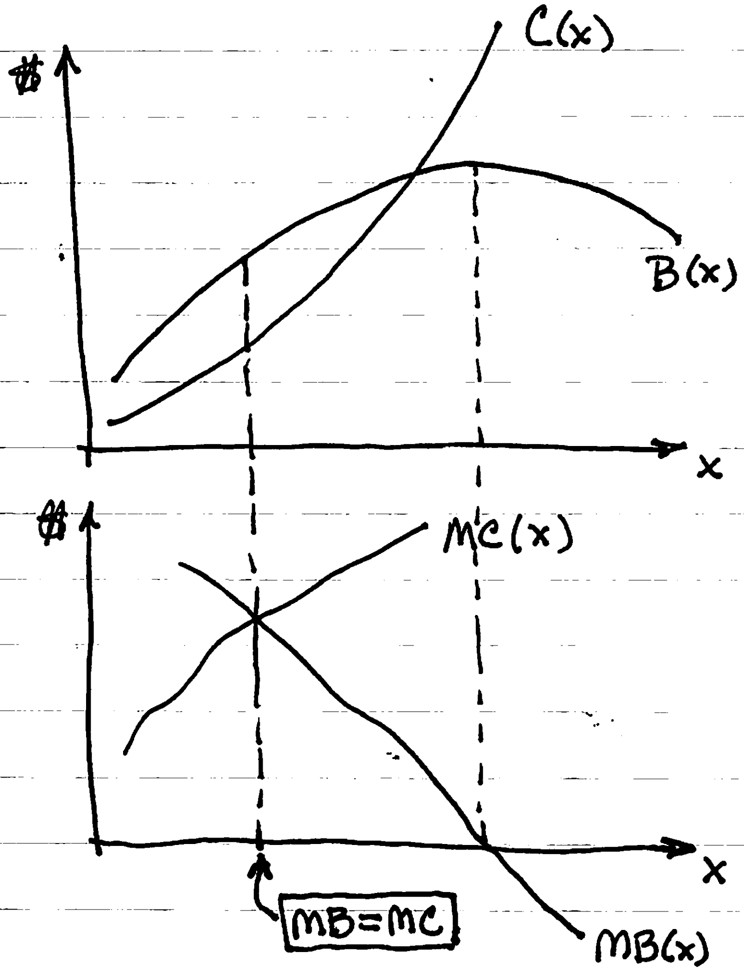
i.e., "MC CUTS MR FROM BELOW"

A SUFFICIENT SECOND-ORDER CONDITION IS THAT
MC IS INCREASING
MR IS DECREASING

IN THE BASIC MODEL, THE PRODUCT IS HOMOGENEOUS AND IS SOLD AT A UNIFORM PRICE. THEREFORE, $R = pq$, SO THE RELATION BETWEEN p AND q — THE DEMAND CURVE THE FIRM FACES — IS CENTRAL.

MARGINAL COST-BENEFIT ANALYSIS

(WITH DIFFERENTIABLE COST AND BENEFIT FUNCTIONS)



MB = MC (1ST-ORDER CONDITION)

"MC CUTS MB FROM BELOW" (2ND-ORDER CONDITION)

NET BENEFIT, $B(x)-C(x)$, CONCAVE.

SUFFICIENT: B CONCAVE AND C CONVEX.

EXAMPLES (SPECIAL CASES):

FIRM: $\pi(x) = R(x) - C(x)$ $\left\{ \begin{array}{l} B(x) \text{ IS } R(x) \\ C(x) \text{ IS } C(x) \end{array} \right.$

CONSUMER: (WITH QUASILINEAR UTILITY FUNCTION)

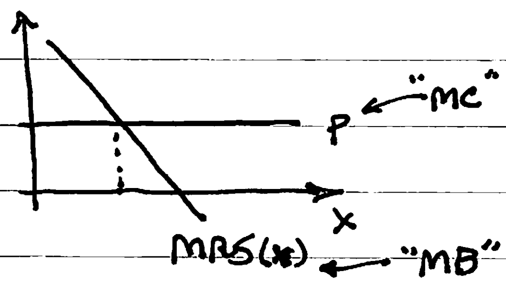
$u(x,y) = y + v(x)$; $MRS = v'(x)$

BUDGET CONSTRAINT: $px + y = w$

$B(x) = v(x)$, $MB(x) = v'(x) = MRS$

$C(x) = px$, $MC(x) = p$

$\therefore$ $MRS = p$ IS 1ST-ORDER CONDITION, $MB = MC$.



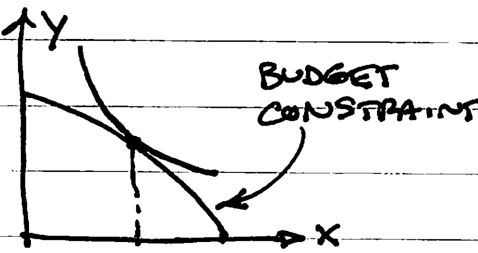
SUPPOSE THE BUDGET CONSTRAINT IS NOT LINEAR

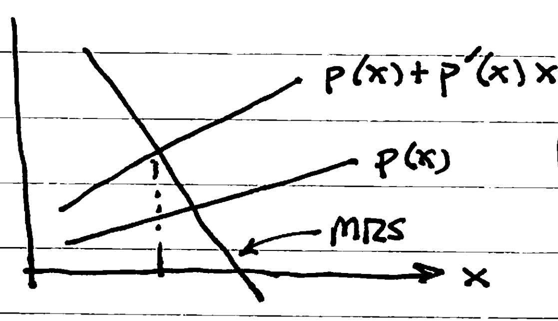
— e.g., p INCREASES AS x INCREASES:

$p(x)x + y = w$

THEN $C(x) = p(x)x$

$MC(x) = p(x) + p'(x)x$





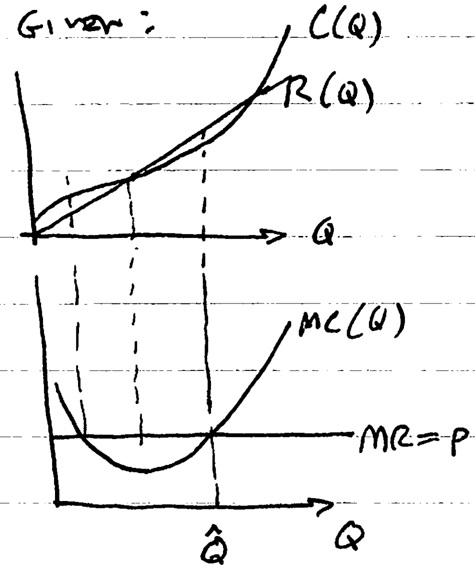
$MRS = p(x) + p'(x)x$

Some Special Cases

① Firm takes price as given:

$$R(Q) = pQ$$

$\therefore MR = p$



"Firm has no market power"

② Demand curve facing firm is linear:

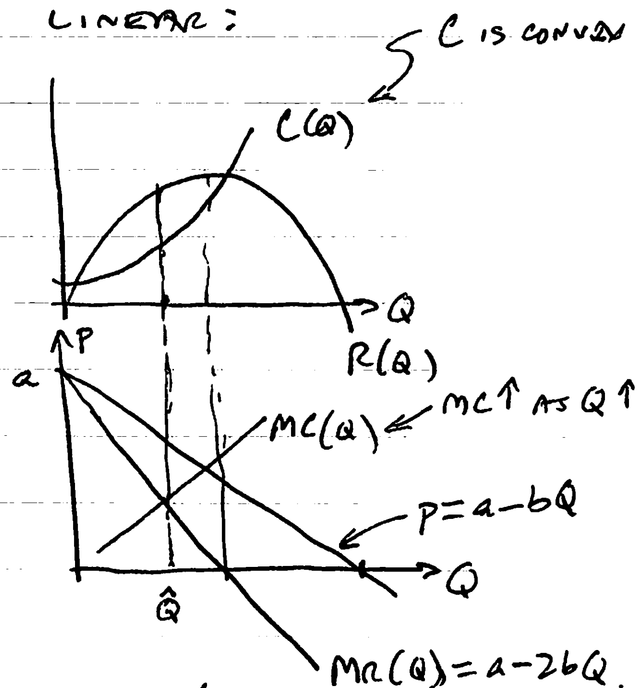
$$R(Q) = pQ = (a-bQ)Q$$
$$= aQ - bQ^2$$

↑

$p = a - bQ$

$MR(Q) = a - 2bQ$,

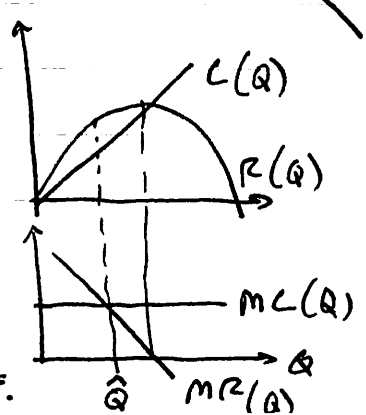
★ { Twice the slope of demand, same vertical intercept a



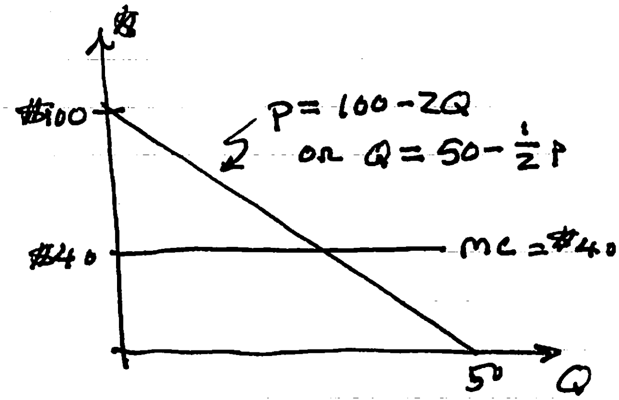
If CRS ($C(Q)$ linear):

$C(Q) = cQ$; $MC = c$

You first encounter this as the model of a monopoly. But it is really the basic model of any firm that has "market power" — a downward-sloping demand curve.



MONOPOLY EXAMPLE



THE DEMAND CURVE/FUNCTION FOR THE FIRM'S PRODUCT:
ASSUME $p = 100 - 2Q$.

THE FIRM'S COST FUNCTION:
ASSUME $C(Q) = 40Q$.

PROFIT = REVENUE − COST: $\pi(Q) := R(Q) - C(Q)$.

Q MAXIMIZES PROFIT AT $\pi'(Q) = 0$ $[\text{IF } \pi''(Q) < 0]$.

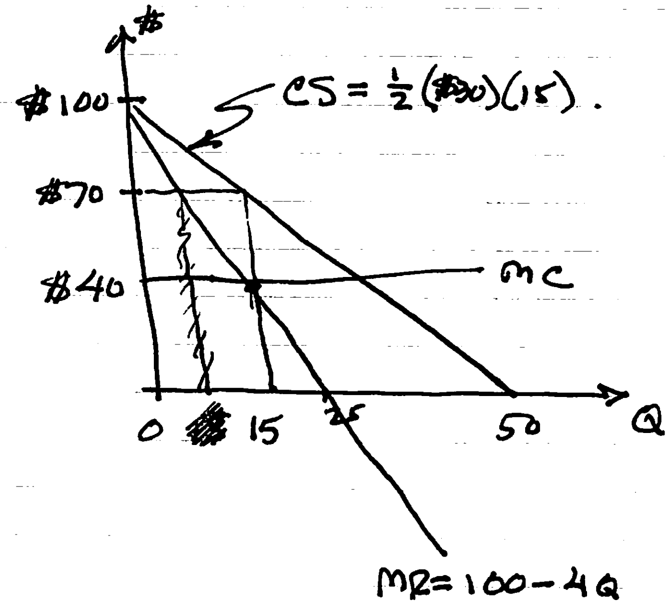
i.e., $R'(Q) - C'(Q) = 0$

i.e., $\boxed{MR(Q) = MC(Q)}$ $[\&\ MR'(Q) < MC'(Q)]$ "MC CUTS MR FROM BELOW"

REVENUE: $R(Q) = pQ = (100 - 2Q)Q = 100Q - 2Q^2$.

$MR(Q) = R'(Q) = 100 - 4Q$

COST: $MC(Q) = C'(Q) = 40$



$MR = MC$: $100 - 4Q = 40$

i.e., $4Q = 60$

i.e., $Q = 15$, $p = \$70$.

$R = \$1050$, $C = \$600$,
$\pi = \$450$

$PS = \pi = \$450$
$CS = \frac{1}{2}(\$30)(15) = \$225$

TOTAL SURPLUS $= \$675$.

COMPETITIVE OUTCOME: $p = MC = \$40$; $Q = 30$; $\pi = \$0$; $CS = \frac{1}{2}(\$60)(30) = \$900$.
$R = \$1200$, $C = \$1200$, $\pi = \$0$.

WE CAN INSTEAD USE P AS THE DECISION VARIABLE:

$$Q = 50 - \tfrac{1}{2}p \qquad \tilde{R}(p) = pQ = \left(50 - \tfrac{1}{2}p\right)p = 50p - \tfrac{1}{2}p^2$$

$$\tilde{C}(p) = C(Q(p)) = 40\,Q(p) = (40)\left(50 - \tfrac{1}{2}p\right) = 2000 - 20p$$

$$\begin{aligned}\pi(p) &= \tilde{R}(p) - \tilde{C}(p)\\ &= \left(50p - \tfrac{1}{2}p^2\right) - (2000 - 20p)\\ &= 70p - \tfrac{1}{2}p^2 - 2000\end{aligned}$$

$$\left.\begin{aligned}\pi'(p) = 70 - p = 0 \text{ AT } p^* &= 70\\ Q &= 15\end{aligned}\right\} \text{ SAME AS WHEN WE USED } Q.$$

BUT NOTE THAT IT'S NOT $MR = MC$ ANY LONGER, BECAUSE THEY WERE FUNCTIONS OF Q.

WE COULD WRITE $M\tilde{R}(p)$ AND $M\tilde{C}(p)$:

$$M\tilde{R}(p) = 50 - p, \quad M\tilde{C}(p) = -20$$

$$M\tilde{R}(p) = M\tilde{C}(p): \; 50 - p = -20,$$

$$\therefore\; p = 70, \; Q = 15.$$