Economics 501B Final Exam
University of Arizona
Fall 2010

1. Alan and Ben each have income today of \$10 per hour. If growth is High by the time tomorrow arrives, Alan's income will still be \$10 per hour, but Ben's income will be \$15 per hour. If growth is Low, Alan's income tomorrow will be only \$8 per hour and Ben's will be only \$12 per hour. Alan's and Ben's preferences are described by the utility functions

$$u^A(x_0, x_H, x_L) = x_0 + 4\sqrt{x_H} + \sqrt{x_L}$$

and

$$u^B(x_0, x_H, x_L) = x_0 + 3\sqrt{x_H} + 2\sqrt{x_L},$$

where x_0 denotes the individual's spending today, x_H denotes his spending tomorrow if growth is High, and x_L denotes his spending tomorrow if growth is Low (all measured per hour).

(a) Determine the interior Pareto efficient allocation(s). You needn't derive the allocation(s) directly from the Pareto definition or from the solutions of a Pareto maximization problem if you can instead obtain the allocation(s) from the appropriate marginal conditions that characterize the Pareto allocations.

(b) Determine the Arrow-Debreu allocation(s) and prices.

In (c), (d), and (e) you can solve directly, or you can appeal to the complete-markets security pricing formula.

(c) In the Arrow-Debreu market structure, what is the (implicit) interest rate?

(d) Suppose the only securities are shares in the firm Gamma Technologies and shares in the firm Delta Insurance. Each share of Gamma will yield \$3 per hour if growth is High and will obligate the holder to *pay* \$2 per hour if growth is Low. Each share of Delta will yield nothing if growth is High, and will yield \$1 per hour if growth is Low. Determine the equilibrium security prices and Alan's and Ben's holdings of the securities.

(e) Now suppose that only the Gamma security is available, but no other security. Will the equilibrium be Pareto optimal? If so, indicate how robust this result is and determine the equilibrium price of Gamma as well as Alan's and Ben's holdings of Gamma. If the equilibrium will not be Pareto optimal, explain why not.

2. Abby and Bill consume only three goods — mutton, wool, and simoleans. Mutton and wool are produced from simoleans as joint products via a single production process, according to the production function $q_M = q_W = f(z)$, where z denotes the quantity of simoleans used as input and q_M and q_W denote the quantities of mutton and wool produced. Abby and Bill are endowed with $\mathring{x}_{i0}$ simoleans ($i = A, B$) and no one has any endowment of mutton or wool. An *allocation* is a list

$$(z, x_{A0}, x_{B0}, x_{AM}, x_{BM}, x_{AW}, x_{BW}) \in \mathbb{R}^7_+.$$

Abby's and Bill's preferences are described by the utility functions

$$u^A(x_{A0}, x_{AM}, x_{AW}) \quad \text{and} \quad u^B(x_{B0}, x_{BM}, x_{BW}),$$

both of which are strictly increasing, continuously differentiable, and strictly quasiconcave. *You might find it helpful to read part (f) before beginning to solve this problem.*

(a) Write down a parametric family of maximization problems the solutions of which are the Pareto allocations.

(b) Determine the first-order marginal conditions (FOMC) that characterize the *interior solutions* of the maximization problems, and convert the FOMC into economic marginal conditions for Pareto efficiency, relating marginal rates of substitution and marginal cost or marginal productivity.

Henceforth assume that $f(z) = \frac{1}{c}z$.

(c) Explain why a price-taking profit-maximizing firm with this constant-returns-to-scale production function must earn profit equal to zero in a market equilibrium.

(d) Denote the market prices by p_0, p_M, and p_W, and assume that $p_0 = 1$ always. Determine the first-order marginal conditions that characterize the utility-maximizing choices of the consumers and the profit-maximizing choices of the firm(s), assuming they are all price-takers.

(e) Combining (b) and (d), show that a market equilibrium is Pareto optimal — *i.e.*, that the conditions for equilibrium, in (d), imply the conditions for Pareto efficiency, in (b).

(f) If we write $c = \frac{1}{1+r}$ — *i.e.*, $f(z) = (1 + r)z$ — then the conditions you obtained in (d) should coincide exactly with the equilibrium conditions we obtained when consumers were uncertain today which of two alternative states of the world would hold when tomorrow arrives (say, state M or state W), and the only available market was a credit market. But here the equilibrium is Pareto optimal, while in the uncertainty case, with only a credit market, the equilibrium was *not* Pareto optimal. Explain why we get these two seemingly opposite results.

3. Amy's and Bev's preferences are both described by the utility function $u(x, y) = \sqrt{x} + \sqrt{y}$. Amy owns the bundle $(\mathring{x}_A, \mathring{y}_A) = (0, 36)$, Bev owns the bundle $(\mathring{x}_B, \mathring{y}_B) = (36, 0)$.

(a) Determine a Walrasian equilibrium allocation and price list. You needn't do this by deriving the equilibrium, but you need to verify that what you have is an equilibrium — *i.e.*, you need to explicitly verify that your allocation and price list satisfy the definition of a Walrasian equilibrium.

(b) Determine the utility frontier for this two-consumer economy.

Your answer to (b) might be helpful in answering the following questions, but the questions can be answered without having an answer for (b).

(c) Determine the *exact* set of core allocations and draw the set in an Edgeworth box diagram.

Now suppose that Amy and Bev are joined by a third person, Cay, who has the same preference as the others, but who owns the bundle (12,12).

(d) Determine a Walrasian equilibrium for this three-person economy. Again, you only have to verify that the equilibrium you've identified is actually an equilibrium.

(e) Determine the set of core allocations in this three-person economy. Describe how the core has changed with the addition of the third person, Cay.