Economics 501B Midterm Exam
University of Arizona
Fall 2011

Problems #1 and #2 should be relatively easy. Be sure to leave enough time for Problem #3, which will require more time.

1. Altogether, Ann and Bob own 30 bottles of beer and 20 bags of peanuts. Using x to denote bottles of beer and y to denote bags of peanuts, Ann's and Bob's preferences are described by the following utility functions:

$$u_A(x_A, y_A) = x_A^3 y_A \qquad \text{and} \qquad u_B(x_B, y_B) = x_B + 2y_B.$$

(a) Determine all the Pareto efficient allocations and depict them in an Edgeworth box diagram.

(b) Ann owns 30 bottles of beer and 15 bags of peanuts. Bob owns 5 bags of peanuts and no beer. Determine the unique Walrasian equilibrium allocation and all associated equilibrium price-lists. Verify that there are no other equilibrium price-lists except the one(s) you have identified.

2. Bart and Arnie are roommates. Arnie smokes and Bart doesn't. Bart doesn't like Arnie's smoke. Let x denote the number of cigars Arnie smokes per day, and let y_A and y_B denote the number of dollars Arnie and Bart spend on other goods. Each roommate's preference is representable by a utility function, as follows:

$$u_A(x, y_A) = y_A + 6x - \frac{1}{2}x^2 \qquad \text{and} \qquad u_B(x, y_B) = y_B - x^2.$$

Arnie does not have to pay for his cigars — he's the editor of *Cigar Guy* magazine and receives cigars free from the cigar company.

(a) Write down a parametric constrained maximization problem for which the solutions are the Pareto optimal outcomes (x, y_A, y_B). Use this maximization problem to determine the interior Pareto outcomes for Bart and Arnie (the ones for which x, y_A and y_B are all positive).

(b) What are Arnie's and Bart's marginal rates of substitution, MRS_A and MRS_B, between x and y_A (for Arnie) and between x and y_B (for Bart), at the interior Pareto outcomes? Explain why, at any other level of x, there would be a Pareto improvement.

3. There are only two consumers, Ann and Bob. Altogether, they own 66 bushels of oranges and 12 gallons of orange juice (OJ). They also own a machine that can turn oranges into OJ according to the production function $q = 12\sqrt{z}$, where z denotes bushels of oranges used as input to the machine and q denotes gallons of OJ produced. Ann's and Bob's preferences for oranges and OJ are described by the utility functions $u_A(x_A, y_A) = x_A^2 y_A$ and $u_B(x_B, y_B) = x_B y_B^2$, where x_i denotes bushels of oranges consumed by i and y_i denotes gallons of OJ consumed by i.

(a) Determine whether there is a Pareto allocation in which Ann consumes $x_A = 40$ bushels of oranges and Bob consumes $x_B = 10$ bushels of oranges. If there is, determine the remaining details of the allocation — the input and output levels in the production plan and Ann's and Bob's consumption of OJ — and verify that the allocation you've constructed is indeed a Pareto allocation. (You can do that by using the marginal conditions that characterize Pareto allocations; you don't need to derive those conditions.) If there is not such a Pareto allocation, verify that.

(b) Now assume that Ann initially owns the bundle $(\mathring{x}_A, \mathring{y}_A) = (44, 0)$ and Bob initially owns the bundle $(\mathring{x}_B, \mathring{y}_B) = (22, 12)$, and assume that Ann owns the machine that produces OJ (and she therefore earns all the profit generated by using the machine to produce OJ). Verify that there is a Walrasian equilibrium in which the price of oranges is $p_x = \$3$ per bushel and the price of OJ is $p_y = \$2$ per gallon. Determine the equilibrium production plan (*i.e.*, the input and output levels), the profit, Ann's and Bob's consumption bundles, and the amount of oranges and OJ each consumer buys or sells in order to achieve these consumption bundles. Verify that this is indeed a Walrasian equilibrium.

(c) Describe the role the so-called "Fisher Separation Theorem" plays in your answer to (b).

(d) Someone has proposed that we allocate the bundle $(20, 28)$ to Ann and the bundle $(10, 56)$ to Bob. Is this allocation feasible — *i.e.*, is there a production plan that will support it? If not, verify that. If there is such a production plan, what is it? Is the resulting outcome Pareto optimal? If so, verify it by showing that it satisfies the appropriate marginal conditions; if it is not Pareto optimal, find a Pareto improvement.