Economics 501B Final Exam
University of Arizona
Fall 2012

1. Ann, Bob, and Cal are neighboring homeowners. Mosquitoes are a nuisance, and the homeowners would like to hire someone to apply a mosquito control spray. Let x denote the number of gallons that are sprayed each week, and let y_i denote the amount of money (in dollars) homeowner i spends on other goods each week. Every mosquito control company charges \$6 per gallon sprayed. The mosquito spray is a public good to the homeowners: any amount that is sprayed affects all three homeowners. Assume that each homeowner's income is large enough that none of the outcomes in this problem exhausts anyone's income.

The three homeowners' preferences are described by the following utility functions or marginal rates of substitution:

$$\begin{aligned} u_A(x, y_A) &= y_A + 15x - \tfrac{1}{2}x^2, \qquad & MRS_A &= 15 - x \ , \\ u_B(x, y_B) &= y_B + 10x - \tfrac{1}{2}x^2, \qquad & MRS_B &= 10 - x \ , \\ u_C(x, y_C) &= y_C + 5x - \tfrac{1}{2}x^2, \qquad & MRS_C &= 5 - x \ . \end{aligned}$$

(a) Determine the interior Pareto allocations.

(b) Suppose each homeowner simply purchases the amount of spray that he or she wishes to have, taking as given the amount purchased by the other two. Determine each homeowner's reaction function, and then show that there is a single Nash equilibrium. Determine the amounts purchased by each homeowner at the Nash equilibrium.

(c) Find a strict Pareto improvement on the Nash equilibrium that is Pareto optimal — *i.e.*, a Pareto allocation that makes all three homeowners strictly better off than at the Nash equilibrium.

(d) Now suppose three new homeowners arrive in the neighborhood: Abby is exactly like Ann (they have the same preferences); Bill is exactly like Bob; and Carl is exactly like Cal. Because the neighborhood is twice as large, it now takes two gallons of spray to have the same effect as one gallon did previously. The easiest way to model this is to leave the utility functions unchanged and to double the cost: it now costs \$12 per gallon — *i.e.*, \$12 to achieve the effect of one more unit of x in each of the utility functions. How does this change the answers to (a) and (b) — what are the Pareto allocations, and what are the Nash equilibria when the homeowners purchase spray on their own?

2. Suppose half the people in the economy choose according to the utility function

$$u^A(x_0, x_H, x_L) = x_0 + x_H - \frac{1}{40}x_H^2 + x_L - \frac{1}{18}x_L^2$$

and the other half according to the utility function

$$u^B(x_0, x_H, x_L) = x_0 + x_H - \frac{1}{32}x_H^2 + x_L - \frac{1}{30}x_L^2$$

where

x_0 represents consumption "today,"

x_H represents consumption "tomorrow" in event H, and

x_L represents consumption "tomorrow" in event L.

Storage of the consumption good from today until tomorrow is not possible. Each person is endowed with twelve units of the good today. Each type A person will be endowed with $\mathring{x}_H^A = 18$ in state H but nothing in state L. Each type B person will be endowed with $\mathring{x}_L^B = 16$ in state L but nothing in state H.

In your answers, consider only allocations that give all type A people the same consumptions and all type B people the same consumptions, so that you will be able to completely describe an allocation with the six variables $x_0^A, x_H^A, x_L^A, x_0^B, x_H^B$, and x_L^B.

(a) Which allocations are Pareto optimal?

(b) Determine the Arrow-Debreu equilibrium — *i.e.*, the Arrow-Debreu prices and allocation.

(c) Suppose that the only market is a credit market (*i.e.*, a market for borrowing and lending). There are no markets in which one can insure oneself against either of tomorrow's two possible events. Everyone is a price-taker — *i.e.*, they all take the interest rate as given. Will the market equilibrium allocation be Pareto optimal? Explain why your answer is the correct one.

(d) In addition to the credit market in (c), suppose there is another market as well, in which one can buy or sell insurance today against the occurrence of event H. Each unit of insurance that a person purchases is a contract in which the seller of the contract agrees to pay the buyer one unit of consumption tomorrow if event H occurs. Let p denote the market price of the insurance: the buyer pays the seller p units of consumption today for each unit of insurance he purchases. Determine the competitive equilibrium prices (*i.e.*, the interest rate and the price p of insurance) and the equilibrium allocation, as well as the amount each person saves or borrows and the amount of insurance each person purchases or sells.

3. There are only two goods in the economy, widgets and simoleans. There are $n = 1000$ consumers; each consumer has the same preference, described by the utility function $u(x, y) = y + 8x - \frac{1}{2}x^2$, where x is the number of widgets consumed and y the number of simoleans consumed. Note that each consumer's MRS function is $MRS = 8 - x$. Each consumer is endowed with 100 simoleans but no widgets. There are $m = 20$ firms; each one can produce widgets, using simoleans as input. Each firm has the same production function, $q = 10\sqrt{z}$, where z is the quantity of simoleans used as input and q is the resulting quantity of widgets produced.

(a) Suppose the firms are not all producing at the same level — specifically, the consumption-production allocation is $A = \left((x_i, y_i)_{i=1}^{n}, (z_j)_{j=1}^{m}\right)$, and the levels of z_j, and therefore q_j, are not the same for all j. Find a Pareto improvement upon A. (Your Pareto improvement need not be Pareto efficient.)

(b) Assume that each firm is fully owned by a different consumer, who has ownership in any profits his firm earns. Assume that the price of simoleans is fixed at unity, and let p denote the price of widgets. Determine the market demand and supply functions for widgets. Determine a Walrasian (price-taking) equilibrium: the price of widgets, the firms' production levels and profits, and the consumers' consumption bundles. Verify that this is indeed a Walrasian equilibrium.

(c) Using the marginal conditions for Pareto efficiency, verify that the equilibrium allocation in part (b) is Pareto efficient. (Do not appeal to the First Welfare Theorem, but you may use the Pareto marginal conditions without deriving them.)

4. Jerry and Elaine have each purchased a large pizza (12 slices each), but now they realize they have nothing to drink with their pizzas. Kramer has two six-packs of beer (12 bottles), but nothing to eat. They decide to get together in Jerry's apartment for dinner. Each has the same preferences, described by the utility function $u(x, y) = xy$, where x and y denote slices of pizza and bottles of beer.

(a) Derive the utility frontier for each coalition.

(b) Use your result in (a) to determine whether the following allocation is in the core:

$$(x_J, y_J) = (6, 3) \qquad (x_E, y_E) = (2, 1) \qquad (x_K, y_K) = (16, 8).$$

(c) Kramer is studying economics and recalls that core allocations treat identical individuals identically. Jerry and Elaine are identical (*i.e.*, they're identical from our economic perspective: they have the same preferences and the same initial endowment). What does this theorem that Kramer remembers tell us about the answer to (b)?

(d) Determine the set of all core allocations.