

Economics 501B Midterm Exam
University of Arizona
Fall 2012

1. Altogether, Al and Bill own 30 bottles of beer and 20 bags of peanuts. Using x to denote bottles of beer and y to denote bags of peanuts, Al's and Bill's preferences are described by the following utility functions:

$$u_A(x_A, y_A) = x_A y_A \quad \text{and} \quad u_B(x_B, y_B) = x_B y_B^2.$$

- (a) Write down a constrained maximization problem, the solutions of which are the Pareto allocations, and write down the first-order conditions that characterize the solutions.
- (b) Determine the set of all Pareto allocations, expressing y_A , x_B , and y_B as functions of x_A , and depict the set in an Edgeworth box diagram.
- (c) Al owns no beer and 6 bags of peanuts. Bill owns 30 bottles of beer and 14 bags of peanuts. Determine the unique Walrasian equilibrium allocation and all associated equilibrium price-lists. Verify that there are no other equilibrium price-lists except the one(s) you have identified.

2. To begin with, Ann owns 30 quarts of wine and no cheese; and Bob owns 20 pounds of cheese and no wine. These are the only two goods. You observe that after they have traded in a market in which they both take the prices as given (*i.e.*, at a Walrasian equilibrium), Ann ends up with the bundle consisting of 12 quarts of wine and 12 pounds of cheese, and Bob ends up with the bundle consisting of 18 quarts of wine and 8 pounds of cheese. Denote this allocation to Ann and Bob by X .

- (a) All you know about their preferences is that each person's preference can be represented by a locally nonsatiated utility function. Prove that the allocation X is a Pareto allocation of the wine and cheese Ann and Bob owned initially. [You may invoke the First Duality Theorem if you find it useful, but be sure to give a precise statement of the theorem and a clear statement of how you are using it. You do not need to prove it.]
- (b) All you know about Ann's and Bob's preferences is that each one's preference is strictly quasiconcave. Is it possible that their preferences are identical? Verify your answer.

3. There are only two goods, grapes and wine. There is a single production process available, which can transform grapes into wine according to the following production function, in which z denotes the pounds of grapes used as input and $f(z)$ the resulting quarts of wine obtained as output:

$$f(z) = \begin{cases} 0, & \text{if } 0 \leq z \leq 20 \\ z - 20, & \text{if } 20 \leq z \leq 80 \\ 20 + \frac{1}{2}z, & \text{if } z \geq 80. \end{cases}$$

There are ten identical consumers, each with a preference ordering described by the utility function $u(x, y) = xy$, where x and y denote pounds of grapes and quarts of wine consumed. Each consumer owns 12 pounds of grapes; there are no other grapes and there is no other wine except what is produced.

- (a) Draw the set of all the aggregate consumption bundles (x, y) that are feasible.
- (b) Determine all the Pareto optimal production-and-consumption plans in which each consumer receives the same bundle as every other consumer.
- (c) Consider the plan in which $z = 80$; $(x_i, y_i) = (5, 3)$ for $i = 1, 2, 3, 4, 5$; and $(x_i, y_i) = (3, 9)$ for $i = 6, 7, 8, 9, 10$. Find a Pareto optimal allocation (specifying the complete production-and-consumption plan) that makes everyone strictly better off.

For questions (d) and (e), assume there is a single firm that owns the production process and the consumers and the firm all behave “competitively” — *i.e.*, they are all price-takers. Assume that the firm chooses an input-output plan that maximizes its profit and that the consumers share equally in any profit the firm earns.

- (d) Assume that the price of grapes is three dollars per pound and the price of wine is five dollars per quart. What input-output plan will the firm choose, and how much will each consumer demand of each good?
- (e) Verify that there is no Walrasian equilibrium.