Economics 501B Final Exam
University of Arizona
Fall 2013

1. Amy and Bev share a house. They each like flowers, so they've decided to create a flower garden. If we denote the number of flowering plants by x and the amount of money each housemate spends on other goods by y_A and y_B, then their preferences are described by the utility functions $u^A(x, y_A)$ and $u^B(x, y_B)$. The household would have to spend $C(x)$ dollars to obtain x flowering plants. Amy has $\mathring{y}_A$ dollars available to spend on plants and other goods; Bev has $\mathring{y}_B$ dollars.

(a) Write down a maximization problem that characterizes the Pareto efficient allocations and use this problem to derive the Samuelson marginal condition for interior Pareto efficiency,

$$MRS_A + MRS_B = MC.$$

For the remainder of this problem assume that $C(x) = \$20x$ (*i.e.*, each flowering plant costs \$20); that $\mathring{y}_A$ and $\mathring{y}_B$ are each at least \$300; and that

$$u^A(x, y_A) = y_A + 44x - 2x^2 \qquad \text{and} \qquad u^B(x, y_B) = y_B + 36x - x^2.$$

(b) Use the result in (a) to determine the set of interior Pareto efficient allocations (*i.e.*, the ones in which x, y_A, and y_B are all positive).

(c) Determine the Lindahl equilibrium, and verify that what you've found is indeed a Lindahl equilibrium. (The equilibrium is unique, but you needn't show this.)

(d) Suppose Amy purchases q_A flowering plants and Bev purchases q_B, each purchasing the number of plants that maximizes her utility taking as given the other's purchases. Determine Amy's and Bev's reaction functions, and draw their reaction curves in a single diagram. Is there a Nash equilibrium? If so, determine the equilibrium q_A and q_B; if not, explain why not.

(e) If there is an equilibrium in (d), determine a Pareto improvement on it. If there isn't an equilibrium, determine a Pareto improvement on the situation in which $q_A = q_B = 4$.

(f) Would it be correct to make welfare comparisons among alternative outcomes by comparing Amy's and Bev's consumer surplus in the alternative outcomes — for example, comparing the outcomes in (c) and (d) to the outcome you determined in (e) by comparing Amy's consumer surplus in each outcome and Bev's consumer surplus in each outcome? Explain briefly.

2. There are two firms producing low-calorie soft drinks: Uno has only one calorie; Dos has two calories. The demand for the firms' products is given by the two equations

$$q_1 = 30 - 2p_1 + p_2 \qquad \text{and} \qquad q_2 = 30 + p_1 - 2p_2,$$

where p_1 and p_2 are the prices (in dollars) Uno and Dos charge per case of bottles, and q_1 and q_2 are the resulting levels of demand. All production is costless — *i.e.*, $MC \equiv 0$ for each firm. Note that it would be equivalent to describe demand for the soft drinks by using the inverse demand functions

$$p_1 = 30 - \frac{2}{3}q_1 - \frac{1}{3}q_2 \qquad \text{and} \qquad p_2 = 30 - \frac{1}{3}q_1 - \frac{2}{3}q_2.$$

(a) Determine the Cournot equilibrium quantites, prices, and profits, and draw the two firms' reaction functions in a single diagram.

(b) Determine the Bertrand equilibrium prices, quantities, and profits, and draw the two firms' reaction functions in a single diagram.

(c) Suppose the two firms collude, charging prices that maximize the total of the two firms' profits. What prices will they charge, how much will each firm sell, and what will be their profits?

(d) Suppose Uno observes that $p_2 = \$12$ and $q_2 = 18$, and suppose that Uno takes $q_2 = 18$ as given. Determine the residual demand curve for Uno; draw this demand curve; and determine Uno's profit-maximizing price and quantity, p_1 and q_1. Now suppose that Uno instead takes $p_2 = \$12$ as given. Determine the residual demand curve for Uno; draw this demand curve; and determine Uno's profit-maximizing price and quantity, p_1 and q_1.

(e) Explain why the Cournot and Bertrand equilibria are different.

(f) If all consumers have the same demand functions and initial bundles, it can be shown that the market demand function will be the same as if there were just one "representative" consumer. So let's assume the demand system given in the equations above is for such a single representative consumer. Determine a utility function $u(q_0, q_1, q_2)$ that yields the above demand functions, where q_0 is the amount the consumer spends on all other goods.

(g) Determine the consumer surplus and the total surplus in (a), (b), and (c).

3. Electricity production in Smalltown requires that an electric power plant be constructed, maintained, and operated, and it also requires fuel to actually produce the electricity. Every unit of capacity costs k dollars per day and allows continuous production of up to one kilowatt, day and night. Fuel to produce one kilowatt-hour (kwh) of electricity costs c dollars. Thus, to produce q_D kwh per day and q_N kwh per night requires $(q_D + q_N)c + k \max\{q_D, q_N\}$ dollars.

Bart and Arnie are the only residents of Smalltown. Their preferences for electricity consumption are described by utility functions $u^i(x_{Di}, x_{Ni}, y_i)$, $i = A, B$, where x_{Di} and x_{Ni} denote daytime and nighttime kwh of electricity consumption by i, and y_i denotes dollars available for i to spend on other goods.

(a) Write down a parametric family of maximization problems the solutions of which are the Pareto allocations.

(b) For the family of problems in (a), derive the first-order marginal conditions (FOMC) that characterize interior solutions in which $x_{DA}+x_{DB} > x_{NA}+x_{NB}$. Convert the FOMC to conditions expressed in terms of marginal rates of substitution.

Henceforth assume that $k = 2$ and $c = 1$, and that

$$u^A(x_{DA}, x_{NA}, y_A) = y_A + 12x_{DA} - \tfrac{1}{2}x_{DA}^2 + 4x_{NA} - \tfrac{1}{2}x_{NA}^2$$
$$u^B(x_{DB}, x_{NB}, y_B) = y_B + \ 6x_{DB} - \tfrac{1}{2}x_{DB}^2 + 6x_{NB} - \tfrac{1}{2}x_{NB}^2.$$

(c) Determine the interior Pareto allocations, *i.e.*, the day and night electricity production levels and the day and night electricity consumption by Arnie and by Bart.

(d) Electricity in Smalltown is produced by a single electric company (a "natural monopoly"), but the Smalltown government regulates the prices the company can charge. The company is required to use "marginal cost pricing" — *i.e.*, to charge prices equal to its marginal costs, as it would do if it were one of several price-taking firms. Determine the prices p_D and p_N that the company should charge for daytime and nighttime use of electricity. How much electricity will each of the two consumers use during the day and during the night?