Economics 501B Midterm Exam
University of Arizona
Fall 2013

1. The following example appears in the course lecture notes: there are four consumers, each with the utility function $u(x,y) = xy$; each of the odd consumers has the initial endowment $(\mathring{x}^1, \mathring{y}^1) = (\mathring{x}^3, \mathring{y}^3) = (0,1)$, and each of the even consumers has the initial endowment $(\mathring{x}^2, \mathring{y}^2) = (\mathring{x}^4, \mathring{y}^4) = (1,0)$. We considered several proposed allocations, one of which was the allocation $(\widehat{x}^i, \widehat{y}^i)_N$ in which each odd consumer receives the bundle $(\widehat{x}^1, \widehat{y}^1) = (\widehat{x}^3, \widehat{y}^3) = (\frac{1}{6}, \frac{1}{6})$ and each even consumer receives the bundle $(\widehat{x}^2, \widehat{y}^2) = (\widehat{x}^4, \widehat{y}^4) = (\frac{5}{6}, \frac{5}{6})$. We found that the coalition $S = \{1,2,3\}$ can unilaterally improve upon this proposal with the S-allocation $(\,(\widetilde{x}^1, \widetilde{y}^1), (\widetilde{x}^2, \widetilde{y}^2), (\widetilde{x}^3, \widetilde{y}^3)\,) = \big(\,(\frac{1}{8}, \frac{1}{4}), (\frac{3}{4}, \frac{3}{2}), (\frac{1}{8}, \frac{1}{4})\,\big)$.

(a) Is there any offer $((\overline{x}^i, \overline{y}^i))_{S'}$ that Consumer 4 could make to Consumers 1 and 3 in which all three members of the coalition $S' = \{1,3,4\}$ would be better off than they are in the allocation $(\,(\widetilde{x}^i, \widetilde{y}^i)_N$, where we naturally define $(\widetilde{x}^4, \widetilde{y}^4)$ to be the bundle $(\mathring{x}^4, \mathring{y}^4) = (1,0)$? Verify your answer: *i.e.*, either exhibit such an offer, or prove that one doesn't exist.

(b) If Consumer 4 can find an offer $((\overline{x}^i, \overline{y}^i))_{S'}$ for S' as in (a), there would surely be a counteroffer that Consumer 2 could make to Consumers 1 and 3 that would make all three members of the coalition $S = \{1,2,3\}$ better off than they are in $((\overline{x}^i, \overline{y}^i))_N$, where, as before, we define $((\overline{x}^2, \overline{y}^2)$ to be the bundle $(\mathring{x}^2, \mathring{y}^2) = (1,0)$. And so on, perhaps ad infinitum, with the two odd types getting progressively larger utility levels and the two even types getting progressively smaller utility levels. To eliminate this sequence of offers and counteroffers, Consumer 4 might be able to accompany his offer $((\overline{x}^i, \overline{y}^i))_{S'}$ to Consumers 1 and 3 with an additional offer $((\overline{\overline{x}}^i, \overline{\overline{y}}^i))_N$ to all three of the other consumers, in which

- all three members of $S' = \{1,3,4\}$ are better off than they are in $(\widetilde{x}^i, \widetilde{y}^i)_N$,
- Consumer 2 is better off than he would be if S' were to implement $((\overline{x}^i, \overline{y}^i))_{S'}$, and
- no coalition can improve upon $((\overline{\overline{x}}^i, \overline{\overline{y}}^i))_N$, thus eliminating further counteroffers.

Either exhibit such an allocation $((\overline{\overline{x}}^i, \overline{\overline{y}}^i))_N$, and explain why no coalition can improve upon it, or else prove that no such allocation exists.

2. There are two goods and two persons (Amy and Bev) in the economy. Denote Consumer i's consumption of the goods by x_i and y_i. Each consumer's preference is described by the utility function $u^i(x_i, y_i) = x_i y_i$. The economy is endowed with 16 units of the x-good and none of the y-good. There are two firms, each of which can use the x-good as input to produce the y-good. When Firm 1 uses z_1 units of input it produces $q_1 = 2\sqrt{z_1}$ units of output; when Firm 2 uses z_2 units of input, it produces $q_2 = \frac{1}{2}z_2$ units of output.

(a) Determine the set of all Pareto efficient allocations.

(b) Amy and Bev each own 8 units of the x-good, and Amy owns the share θ_A of each firm's profits and Bev owns the share θ_B of each firm's profits. Of course, $\theta_A, \theta_B \geqq 0$ and $\theta_A + \theta_B = 1$. Determine the set of all Walrasian equilibria — the relative prices, the two firms' production plans and profits, and the two consumers' consumption bundles.

3. Al and Bill share an office. They can adjust the temperature in the office, costlessly, to whatever level they wish. (Their employer pays the heating and air conditioning bills.) Al's and Bill's preferences can be represented by the utility functions $u^A(x, y_A)$ and $u^B(x, y_B)$, where x denotes the office temperature (say, in degrees Celsius), and y_i denotes i's consumption of simoleans. An allocation is therefore a triple $(x, y_A, y_B) \in \mathbb{R}^3_+$. Note that Al and Bill must each "consume" the same "quantity" of x: x has no subscript in an allocation or in their utility functions. Assume that each utility function is differentiable and strictly quasiconcave, and that each consumer's MRS is positive for small values of x and negative for large values of x: if it's very cold, each would like it warmer, and if it's very warm, each would like it cooler. Altogether, they own a total of $\mathring{y}$ simoleans.

Show that the marginal condition that characterizes the interior Pareto allocations is *not* $MRS_A = MRS_B$, but is instead $MRS_A + MRS_B = 0$ — *i.e.*, $MRS_A = -MRS_B$. Do this by first characterizing the Pareto allocations as the solutions of a constrained maximization problem, then obtaining the first-order conditions that characterize the solutions of the maximization problem, and then deriving the condition $MRS_A + MRS_B = 0$ from the first-order conditions.