Economics 501B Midterm Exam
University of Arizona
Fall 2014

For Problems #1 and #2, assume there are two goods in the economy and two consumers, Amy and Bev. Denote Amy's and Bev's consumption bundles by (x_A, y_A) and (x_B, y_B). Amy and Bev each own eight units of the x-good and none of the y-good, and they have the same preference over consumption bundles, described by the utility function $u(x, y) = xy$. There are two firms that can use the x-good as input to produce the y-good. When Firm 1 uses z_1 units of the x-good as input, its output is $q_1 = 2\sqrt{z_1}$ units of the y-good; Amy is the sole owner of Firm 1 (all its profits accrue to her). Bev is the sole owner of Firm 2; when Firm 2 uses z_2 units of the x-good as input, its output is $q_2 = \frac{1}{2}z_2$ units of the y-good.

1. Determine the Pareto efficient production plans and consumption bundles.

2. Determine the Walrasian equilibrium, which is unique (except, of course, that only the relative prices are determined).

For the remainder of the exam, assume there are two goods, with quantities denoted by x and y. The total amounts available to Ann and Bob together are $\mathring{x}$ and $\mathring{y}$, and these quantities satisfy $\mathring{x} = \mathring{y}$. Ann's preferences are given by the utility function $u^A(x, y) = \sqrt{xy}$ and Bob's preferences are given by the utility function $u^B(x, y) = \min\{x, y\}$. For Problems #3, 4, and 5, assume that Ann owns all the x-good, $\mathring{x}_A = \mathring{x}$, and Bob owns all the y-good, $\mathring{y}_B = \mathring{y}$.

3. Determine the set of Pareto allocations.

4. Derive the utility frontier for Ann and Bob.

5. Determine the unique Walrasian equilibrium, and verify directly from the "disaggregated" definition of Walrasian equilibrium that the price-list and allocation you've identified are indeed a Walrasian equilibrium.

6. Now suppose Ann's and Bob's initial endowments are $(\mathring{x}_A, \mathring{y}_A) = (\frac{4}{5}, \frac{1}{5})$ and $(\mathring{x}_B, \mathring{y}_B) = (\frac{1}{5}, \frac{4}{5})$. Indicate how your answers to #3,4, and 5 are affected, and then determine the core of this two-person economy. Depict the core in an Edgeworth box diagram.

Now assume that Ann's and Bob's initial endowments are $(\mathring{x}_A, \mathring{y}_A) = (1, 0)$ and $(\mathring{x}_B, \mathring{y}_B) = (0, 1)$, and they're joined by Cal, who owns the bundle $(\mathring{x}_C, \mathring{y}_C) = (1, 1)$ and whose preferences are described by the utility function $u(x, y) = \frac{1}{2}(x + y)$. Note that we have $\mathring{x} = \mathring{y} = 2$.

7. You've already derived the utility frontier for the coalition $S = \{A, B\}$, in #4, above. Now derive the utility frontiers for the coalitions $S = \{A, B, C\}$, $S = \{A, C\}$, and $S = \{B, C\}$.

8. Determine the core allocations for this three-person economy. (This is a little bit more challenging than the other questions.) If you're unable to determine all the core allocations, identify at least *one* core allocation.