Economics 501B Fall 2013 Final Exam Solutions

1. (a) $\max\limits_{x,y_A,y_B} u^A(x,y_A)$ s.t. $x, y_A, y_B \geqq 0$ and to

$$C(x) + y_A + y_B \leqq \mathring{y}_A + \mathring{y}_B$$

$$u(x, y_B) \geqq \overline{u}_B.$$

The interior first-order marginal conditions:

There are Lagrange multipliers $\sigma \geqq 0$ and $\lambda \geqq 0$ such that

$$\begin{aligned} x: &\quad u_x^A = \sigma C'(x) - \lambda u_x^B, \\ y_A: &\quad u_y^A = \sigma, \\ y_B: &\quad 0 = \sigma - \lambda u_y^B, \end{aligned}$$

where u_x^i and u_y^i denote partial derivatives.

The first equation can be written

$$\frac{1}{\sigma}u_x^A + \frac{1}{\sigma}\lambda u_x^B = C'(x),$$

and combining this with the other two equations, we have

$$\frac{u_x^A}{u_y^A} + \frac{u_x^B}{u_y^B} = C'(x), \qquad i.e., \; MRS_A + MRS_B = MC.$$

(b) $MRS_A = 44 - 4x$ and $MRS_B = 36 - 2x$, so $MRS_A + MRS_B = MC$ yields

$$80 - 6x = 20; \quad i.e., \;\; x = 10.$$

We also must have $y_A + y_B = \mathring{y}_A + \mathring{y}_B - 200$, since $C(10) = 200$.

(c) At the Pareto level of $x = 10$, we have $MRS_A = 4$ and $MRS_B = 16$, so the Lindahl prices are $p_A = 4$ and $p_B = 16$. Each person's utility is maximized at $x = 10$. We have $y_A = \mathring{y}_A - 40$ and $y_B = \mathring{y}_B - 160$. (Note that $MRS_i = p_i$ for $i = A, B$ and that $p_Ax + p_Bx = 40 + 160 = 200 = C(10)$.)

(d) Amy's payoff function is $\pi_A(q_A, q_B) = \mathring{y}_A - 20q_A + 44(q_A + q_B) - 2(q_A + q_B)^2$, which is strictly concave in q_A and has first-order condition

$$\frac{\partial \pi_A}{\partial q_A} = 24 - 4(q_A + q_B) \leqq 0 \qquad \text{and} \qquad 24 - 4(q_A + q_B) = 0 \;\text{ if } q_A > 0.$$

Similarly, $\pi_B(q_A, q_B) = \mathring{y}_B - 20q_B + 36(q_A + q_B) - (q_A + q_B)^2$, with first-order condition

$$\frac{\partial \pi_B}{\partial q_B} = 16 - 2(q_A + q_B) \leqq 0 \qquad \text{and} \qquad 16 - 2(q_A + q_B) = 0 \ \text{ if } q_B > 0.$$

The reaction functions are therefore

$$q_A = \begin{cases} 6 - q_B, & q_B \leqq 6 \\ 0, & q_B \geqq 6 \end{cases} \qquad \text{and} \qquad q_B = \begin{cases} 8 - q_A, & q_A \leqq 8 \\ 0, & q_A \geqq 8. \end{cases}$$

See the attached graph of the reaction functions.

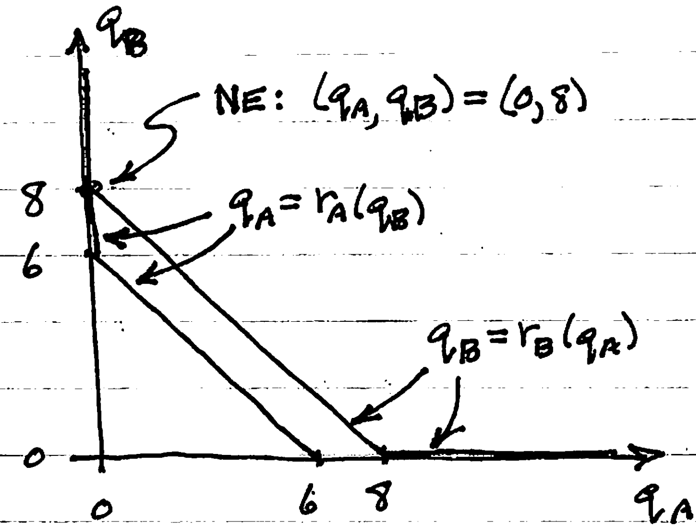
The reaction functions can be written as follows when q_A and q_B are positive:

$$q_A + q_B = 6 \qquad \text{and} \qquad q_A + q_B = 8.$$

Therefore one of the quantities must be zero in equilibrium; clearly, $q_A = 0$ and $q_B = 8$. Thus, $x = 8$ at the Nash equilibrium, less than the Pareto level of $x = 10$.

(e) Increasing x to $x = 9$ or $x = 10$ will yield a Pareto improvement if we allocate the additional \$20-per-unit cost appropriately. Since at $x = 10$ we have $MRS_A = 4$ and $MRS_B = 16$, and each MRS_i is larger than these numbers for all x between 8 and 10, we will increase both utility levels if Amy pays \$4 per unit and Bev pays \$16 per unit for the additional units. This can be verified directly, by comparing their utilities at $x = 8$ and $x = 10$ (including these payments), or equivalently (because of the quasilinear utility functions) comparing consumer surpluses. At the Nash equilibrium we have $CS_A = 224$ and $CS_B = 64$. Increasing x to $x = 10$ with additional payments of \$8 by Amy and \$32 by Bev yields $CS_A = 232$ and $CS_B = 68$. Thus, the "utility gain" to Amy is 8 and to Bev it's 4.

(f) Comparing consumer surplus will yield only an approximation to the correct welfare comparisons in general, but with quasilinear utility functions the comparisons are exact. This follows from the fact that utilities are measured in terms of the good that enters the utility functions linearly. (In this problem, the y-units are actually dollars, so consumer surplus and utility are both measured in dollars.)



q_B
NE: $(q_A, q_B) = (0, 8)$
8
6
$q_A = r_A(q_B)$
$q_B = r_B(q_A)$
0
0
6
8
q_A

② (a) $\pi_1(q_1, q_2) = R_1(q_1, q_2) = 30q_1 - \frac{2}{3}q_1^2 - \frac{1}{3}q_2 q_1$

$$\frac{\partial \pi_1}{\partial q_1} = 30 - \frac{4}{3}q_1 - \frac{1}{3}q_2 = 0 \text{ IF } \frac{4}{3}q_1 + \frac{1}{3}q_2 = 30$$

i.e., $4q_1 + q_2 = 90$

SYMMETRICALLY, FOMC$_2$ IS $q_1 + 4q_2 = 90$

$\therefore$ NE IS $q_1 = q_2 = 18$ AND $p_1 = p_2 = \$12$.

$\pi_1 = \pi_2 = (\$12)(18) = \$216$.

(b) $\pi_1(p_1, p_2) = R_1(p_1, p_2) = 30p_1 - 2p_1^2 + p_2 p_1$

$$\frac{\partial \pi_1}{\partial p_1} = 30 - 4p_1 + p_2 = 0 \Leftrightarrow 4p_1 - p_2 = 30$$

SYMMETRICALLY, FOMC$_2$ IS $4p_2 - p_1 = 30$.

$\therefore$ NE IS $p_1 = p_2 = \$10$ AND $q_1 = q_2 = 20$. $\pi_1 = \pi_2 = \$200$.

(c) $\pi(q_1, q_2) = \pi_1(q_1, q_2) + \pi_2(q_1, q_2)$

$$= 30q_1 - \frac{2}{3}q_1^2 - \frac{1}{3}q_2 q_1 + 30q_2 - \frac{2}{3}q_2^2 - \frac{1}{3}q_1 q_2$$

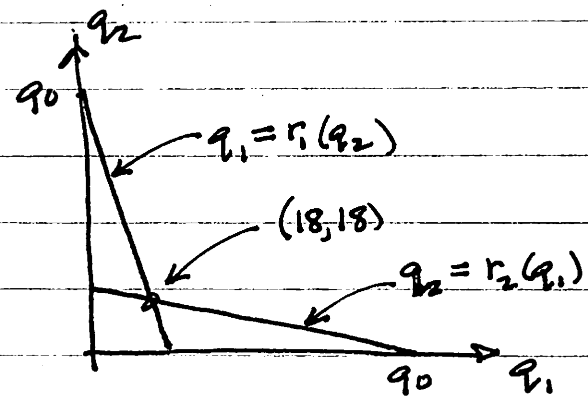
$$\frac{\partial \pi}{\partial q_1} = 30 - \frac{4}{3}q_1 - \frac{1}{3}q_2 - \frac{1}{3}q_2 = 30 - \frac{4}{3}q_1 - \frac{2}{3}q_2$$

$= 0 \Leftrightarrow 4q_1 + 2q_2 = 90$, i.e., $2q_1 + q_2 = 45$.

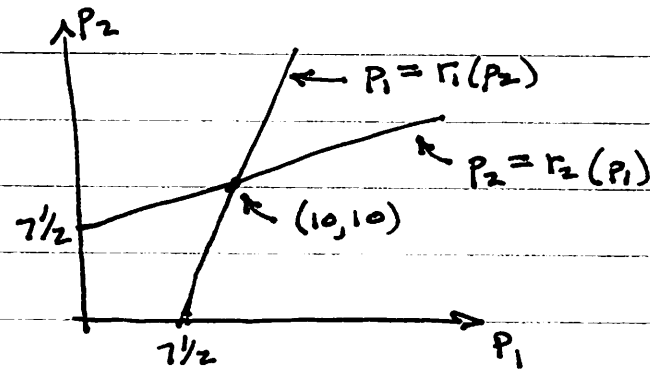
SYMMETRICALLY, $\frac{\partial \pi}{\partial q_2} = 0 \Leftrightarrow q_1 + 2q_2 = 45$.

$\therefore q_1 = q_2 = 15$ AND $p_1 = p_2 = \$15$. $\pi_1 + \pi_2 = \$450$.

COURNOT REACTION FUNCTIONS:



BERTRAND REACTION FUNCTIONS:



(d) SUPPOSE $P_2 = \$12$ AND $q_2 = 18$, AS IN THE COURNOT EQUIL'M:

SUPPOSE FIRM 1 TAKES $q_2 = 18$ AS GIVEN:

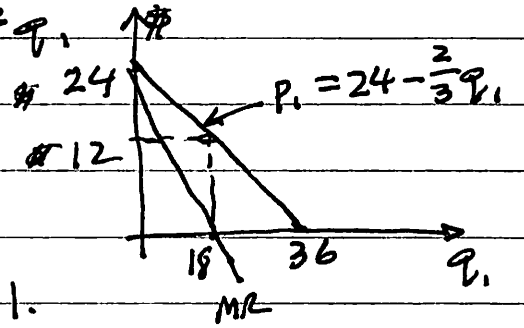
THE RESIDUAL DEMAND CURVE FIRM 1 FACES IS

$$P_1 = 30 - \frac{2}{3}q_1 - 6 = 24 - \frac{2}{3}q_1$$

$$MR_1 = 24 - \frac{4}{3}q_1$$

$= 0$ AT $q_1 = 18$, $P_1 = \$12$,

THE COURNOT EQ'M DECISION FOR FIRM 1.



SUPPOSE FIRM 1 TAKES $P_2 = \$12$ AS GIVEN:

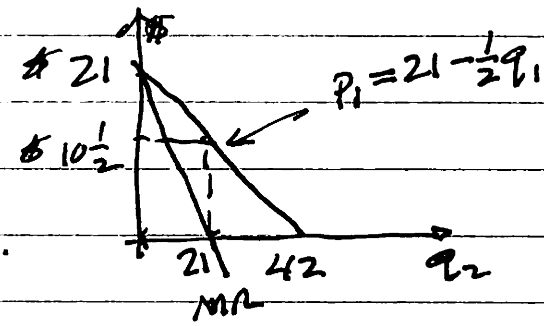
THE RESIDUAL DEMAND CURVE FIRM 1 FACES IS

$q_1 = 42 - 2P_1$; i.e.,

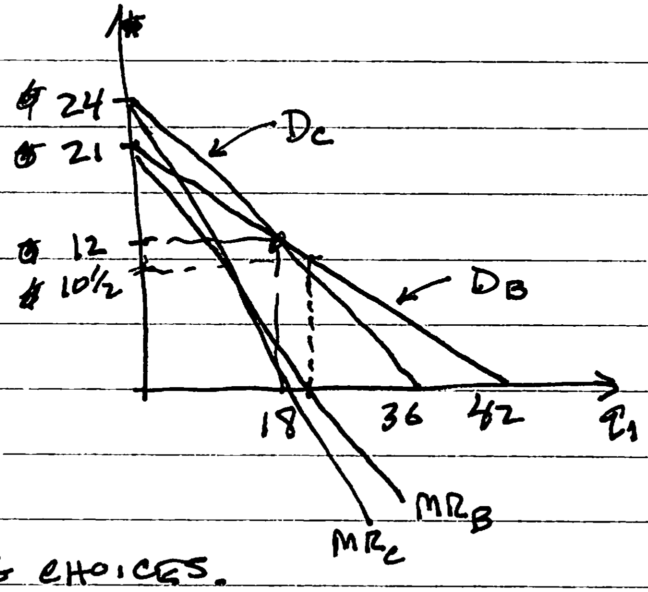
$$P_1 = 21 - \frac{1}{2}q_1$$

$$MR_1 = 21 - q_1$$

$= 0$ AT $q_1 = 21$, $P_1 = \$10\frac{1}{2}$.



(e) THE TWO EQUILIBRIA MAKE DIFFERENT ASSUMPTIONS ABOUT WHAT A FIRM TAKES AS GIVEN, SO THAT THE RESIDUAL DEMAND CURVE A FIRM FACES IS DIFFERENT IN THE TWO CASES (AND ∴ THE MR IS ALSO DIFFERENT). THIS RESULTS IN DIFFERENT PROFIT-MAXIMIZING CHOICES.



(f) FIND A FUNCTION $u(q) = u(q_0, q_1, q_2)$ FOR WHICH

$$MRS_1 = 30 - \frac{2}{3}q_1 - \frac{1}{3}q_2, \quad \text{WHERE } MRS_1 = \frac{u_1}{u_0}$$

$$MRS_2 = 30 - \frac{1}{3}q_1 - \frac{2}{3}q_2, \quad \text{WHERE } MRS_2 = \frac{u_2}{u_0}.$$

WE MUST HAVE (EXCEPT FOR A MONOTONE TRANSFORM)

$$u(q) = 30q_1 - \frac{1}{3}q_1^2 - \frac{1}{3}q_1q_2 + q_0 + f(q_2)$$

AND $u(q) = 30q_2 - \frac{1}{3}q_2^2 - \frac{1}{3}q_1q_2 + q_0 + g(q_1)$

FOR SOME FUNCTIONS f AND g. A SOLUTION IS

$$f(q_2) = 30q_2 - \frac{1}{3}q_2^2 \quad \text{AND} \quad g(q_1) = 30q_1 - \frac{1}{3}q_1^2,$$

AND THEREFORE

$$u(q_0, q_1, q_2) = 30q_1 - \frac{1}{3}q_1^2 + 30q_2 - \frac{1}{3}q_2^2 - \frac{1}{3}q_1q_2 + q_0$$

$$= q_0 + 30(q_1 + q_2) - \frac{1}{3}(q_1^2 + q_2^2 + q_1q_2).$$

(g) $CS = u(q) - u(\overset{\circ}{q}) = u(q_0, q_1, q_2) - u(\overset{\circ}{q}_0, 0, 0)$

$$= q_0 - \overset{\circ}{q}_0 + 30(q_1 + q_2) - \frac{1}{3}(q_1^2 + q_2^2 + q_1q_2)$$

$$= -p_1q_1 - p_2q_2 + 30(q_1 + q_2) - \frac{1}{3}(q_1^2 + q_2^2 + q_1q_2)$$

IN (a): $CS = -(12)(18) - (12)(18) + 30(36) - \frac{1}{3}(324 + 324 + 324)$

$$= -432 + 1080 - 324 = \$324$$

$CS + PS = \$324 + \$432 = \$756.$

IN (b): $CS = -(10)(20) - (10)(20) + 30(40) - \frac{1}{3}(400 + 400 + 400)$

$$= -400 + 1200 - 400 = \$400$$

$CS + PS = \$400 + \$400 = \$800.$

IN (c): $CS = -(15)(15) - (15)(15) + 30(30) - \frac{1}{3}(225 + 225 + 225)$

$$= -450 + 900 - 225 = \$225$$

$CS + PS = \$225 + \$450 = \$675.$

Peak/Off-Peak Problem: Solution

(3)

(a) $\max\limits_{(x_{Di}, x_{Ni}, y_i)_1^2,\, q} \lambda_A u^A(x_{DA}, x_{NA}, y_A)$ s.t. — Lagrange mult'rs:

$u^B(x_{DB}, x_{NB}, y_B) \geqq \bar{u}^B$ — λ_B

$x_{DA} + x_{DB} \leqq q$ — σ_D

$x_{NA} + x_{NB} \leqq q$ — σ_N

$c(x_{DA} + x_{DB} + x_{NA} + x_{NB}) + kq + y_A + y_B \leqq \mathring{y}$ — σ_y

(b) FOMC: (interior)

$$i = A, B \quad \begin{cases} x_{Di}: & \lambda_i u_{Di} = \sigma_D + c\sigma_y \\ x_{Ni}: & \lambda_i u_{Ni} = \sigma_N + c\sigma_y \\ y_i: & \lambda_i u_{yi} = \sigma_y \\ q: & k\sigma_y = \sigma_D + \sigma_N \end{cases}$$

Since $x_{DA} + x_{DB} > x_{NA} + x_{NB}$, we have $x_{NA} + x_{NB} < q$, and therefore $\sigma_N = 0$. Consequently, $u_{Ni} = c\sigma_y$ and $k\sigma_y = \sigma_D$.

Combining these equations:

$$\left.\begin{aligned} \lambda_i u_{Di} &= (k+c)\sigma_y \\ \lambda_i u_{Ni} &= c\sigma_y \\ u_{yi} &= \sigma_y \end{aligned}\right\} \quad \frac{u_{Di}}{u_{yi}} = k+c \text{ and } \frac{u_{Ni}}{u_{yi}} = c$$

i.e., $MRS_D^i = k + c$ and $MRS_N^i = c$, $i = A, B$.

(c)
$$\left.\begin{aligned} MRS_D^A &= 12 - x_{DA} = 3; \text{ i.e., } x_{DA} = 9 \\ MRS_D^B &= 6 - x_{DB} = 3; \text{ i.e., } x_{DB} = 3 \end{aligned}\right\} X_D = 12$$

$$\left.\begin{aligned} MRS_N^A &= 4 - x_{NA} = 1; \text{ i.e., } x_{NA} = 3 \\ MRS_N^B &= 6 - x_{NB} = 1; \text{ i.e., } x_{NB} = 5 \end{aligned}\right\} X_N = 8$$

$\therefore$ Plant size (capacity) is $q = 12$.

(d) Prices are $p_D = k + c = 3$ and $p_N = c = 1$. Electricity usage is as in (c).

$\mathring{y}_A + \mathring{y}_B - (y_A + y_B) = 44 = C(x_D = 12, x_N = 8)$

$y_A = \mathring{y}_A - (3)(9) - (1)(3) = \mathring{y}_A - 30$; $y_B = \mathring{y}_B - (3)(3) - (1)(5) = \mathring{y}_B - 14$.