CHAPTER 4

PARTIAL EQUILIBRIUM

In previous chapters, we studied the behavior of individual consumers and firms, describing optimal behavior when market prices were fixed and beyond the agent's control. Here we begin to explore the consequences of that behavior when consumers and firms come together in markets. First, we'll consider price and quantity determination in a single market or group of closely related markets. Then we'll assess those markets from a social point of view. Along the way, we'll pay special attention to the close relationship between a market's competitive structure and its social "performance."

4.1 PERFECT COMPETITION

In perfectly competitive markets, buyers and sellers are sufficiently large in number to ensure that no single one of them, alone, has the power to determine market price. Buyers and sellers are price takers, and each decides on a self-interested course of action in view of individual circumstances and objectives. A buyer's demand for any one good is, as we've seen, the outcome of a larger utility-maximizing plan over all goods subject to the budget constraint. Similarly, a seller's supply of that good is the outcome of an overall profit-maximizing plan subject to the selling price of that good, technological possibilities and input prices. Equilibrium in a competitive market thus requires the simultaneous compatibility of the disparate and often conflicting self-interested plans of a large number of different agents.

The demand side of a market is made up of all potential buyers of the good, each with their own preferences, consumption set, and income. We let $\mathcal{I} \equiv \{1, \ldots, I\}$ index the set of individual buyers and $q^i(p, \mathbf{p}, y^i)$ be i's nonnegative demand for good q as a function of its own price, p, income, y^i, and prices, $\mathbf{p}$, for all other goods. Market demand for q is simply the sum of all buyers' individual demands

$$q^d(p) \equiv \sum_{i \in \mathcal{I}} q^i(p, \mathbf{p}, y^i). \tag{4.1}$$

There are several things worth noting in the definition of market demand. First, $q^d(p)$ gives the total amount of q demanded by all buyers in the market. Second, because each buyer's demand for q depends not only on the price of q, but on the prices of all other goods as well, so, too, does the market demand for q, though we will generally suppress explicit mention of this. Third, whereas a single buyer's demand depends on the level of her own income, market demand depends both on the *aggregate level* of income in the market and on its *distribution* among buyers. Finally, because individual demand is homogeneous of degree zero in all prices and the individual's income, market demand will be homogeneous of degree zero in all prices and the *vector* of buyers' incomes. Although several restrictions on an individual's demand system follow from utility maximization, homogeneity is the *only* such restriction on the market demand for a single good.

The supply side of the market is made up of all potential sellers of q. However, we sometimes distinguish between firms that are potential sellers in the short run and those that are potential sellers in the long run. Earlier, we defined the short run as that period of time in which at least one input (for example, plant size) is fixed to the firm. Consistent with that definition, in the short-run market period, the number of potential sellers is fixed, finite, and limited to those firms that "already exist" and are in some sense able to be up and running simply by acquiring the necessary variable inputs. If we let $\mathcal{J} \equiv \{1, \ldots, J\}$ index those firms, the **short-run market supply function** is the sum of individual firm short-run supply functions $q^j(p, \mathbf{w})$:

$$q^s(p) \equiv \sum_{j \in \mathcal{J}} q^j(p, \mathbf{w}). \tag{4.2}$$

Market demand and market supply together determine the price and total quantity traded. We say that a competitive market is in **short-run equilibrium** at price p^* when $q^d(p^*) = q^s(p^*)$. Geometrically, this corresponds to the familiar intersection of market supply and market demand curves drawn in the (p, q) plane. Note that by construction of market demand and market supply, market equilibrium is characterized by some interesting and important features: Each price-taking buyer is buying her optimal amount of the good at the prevailing price, and each price-taking firm is selling its profit-maximizing output at the same prevailing price. Thus, we have a true equilibrium in the sense that no agent in the market has any incentive to change his behavior—each is doing the best he can under the circumstances he faces.

EXAMPLE 4.1 Consider a competitive industry composed of J identical firms. Firms produce output according to the Cobb-Douglas technology, $q = x^{\alpha} k^{1-\alpha}$, where x is some variable input such as labor, k is some input such as plant size, which is fixed in the short run, and $0 < \alpha < 1$. In Example 3.6, we derived the firm's short-run profit and supply functions with this technology. At prices p, w_x, and w_k, maximum profits are

$$\pi^j = p^{1/1-\alpha} w_x^{\alpha/\alpha-1} \alpha^{\alpha/1-\alpha}(1-\alpha)k - w_k k, \tag{E.1}$$

and output supply is

$$q^j = p^{\alpha/1-\alpha} w_x^{\alpha/\alpha-1} \alpha^{\alpha/1-\alpha} k. \tag{E.2}$$

If $\alpha = 1/2$, $w_x = 4$, and $w_k = 1$, then supposing each firm operates a plant of size $k = 1$, firm supply reduces to $q^j = p/8$. The market supply function with $J = 48$ firms will be

$$q^s = 48(p/8) = 6p. \tag{E.3}$$

Let market demand be given by

$$q^d = 294/p. \tag{E.4}$$

We can use (E.1) through (E.4) to solve for the short-run equilibrium price, market quantity, output per firm, and firm profits:

$$\begin{aligned} p^* &= 7, \\ q^* &= 42, \\ q^j &= 7/8, \\ \pi^j &= 2.0625 > 0. \end{aligned}$$

This equilibrium, at both market and individual firm levels, is illustrated in Fig. 4.1. (Note that short-run cost curves for firms with this technology can be derived from Exercise 3.34.) □

In the long run, no inputs are fixed for the firm. Incumbent firms—those already producing—are free to choose optimal levels of all inputs, including, for example, the size of their plant. They are also free to leave the industry entirely. Moreover, in the long run, *new* firms may decide to begin producing the good in question. Thus, in the long run, there are possibilities of **entry** and **exit** of firms. Firms will enter the industry in response to positive long-run economic profits and will exit in response to negative long-run profits (losses).

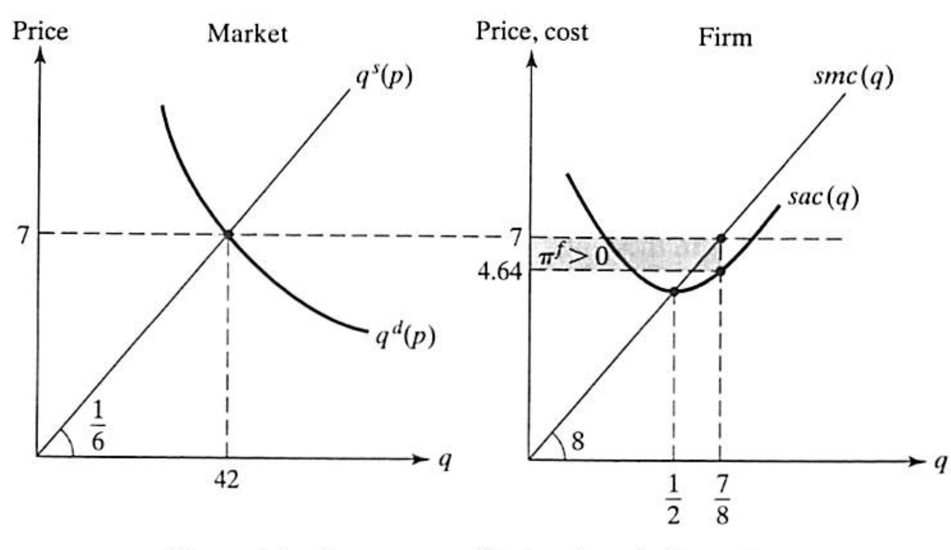

Figure 4.1. Short-run equilibrium in a single market.

In a long-run equilibrium, we shall require not only that the market clears but also that no firm has an incentive to enter or exit the industry. Clearly, then, long-run profits must be nonnegative; otherwise, firms in the industry will wish to exit. On the other hand, because all firms have free access to one another's technology (in particular, firms currently not producing have access to the technology of every firm that is producing), no firm can be earning positive profits in the long run. Otherwise, firms outside the industry will adopt the technology of the firm earning positive profits and enter the industry themselves.

Thus, *two* conditions characterize long-run equilibrium in a competitive market:

$$q^d(\hat{p}) = \sum_{j=1}^{\hat{J}} q^j(\hat{p}),$$
$$\pi^j(\hat{p}) = 0, \qquad j = 1, \ldots, \hat{J}. \tag{4.3}$$

The first condition simply says the market must clear. The second says long-run profits for all firms in the industry must be zero so that no firm wishes to enter or exit the industry. In the short run, the number of firms is given and the market-clearing condition determines the short-run equilibrium price. Note, however, that in the long run, the market-clearing and zero-profit conditions together determine both long-run equilibrium price *and* the long-run equilibrium number, $\hat{J}$, of firms in the industry.

EXAMPLE 4.2 Let inverse market demand be the linear form

$$p = 39 - 0.009q. \tag{E.1}$$

Technology for producing q is identical for all firms, and all firms face identical input prices. The long-run profit function for a representative firm is given by

$$\pi^j(p) = p^2 - 2p - 399, \tag{E.2}$$

so that its output supply function is

$$y^j = \frac{d\pi(p)}{dp} = 2p - 2. \tag{E.3}$$

Note that $y^j \geq 0$ requires $p \geq 1$.

In the long run, market-equilibrium price $\hat{p}$ and the equilibrium number of firms $\hat{J}$ must satisfy the two conditions (4.3). Thus, we must have

$$(1000/9)(39 - \hat{p}) = \hat{J}(2\hat{p} - 2),$$
$$\hat{p}^2 - 2\hat{p} - 399 = 0.$$

From the zero-profit condition, we obtain $\hat{p} = 21$. Substituting into the market-clearing condition gives $\hat{J} = 50$. From (E.3), each firm produces an output of 40 units in long-run equilibrium. This market equilibrium is illustrated in Fig. 4.2. □

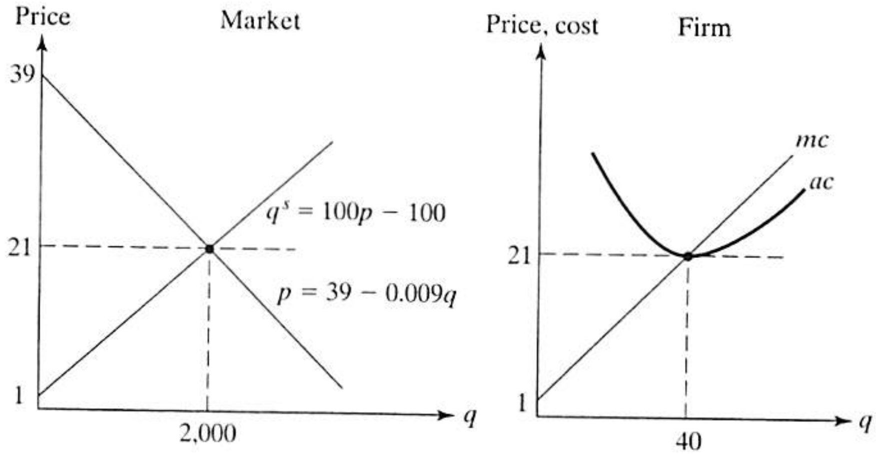

Figure 4.2. Long-run equilibrium in a competitive market.

EXAMPLE 4.3 Let's examine long-run equilibrium in the market of Example 4.1. There, technology was the constant-returns-to-scale form, $q = x^{\alpha} k^{1-\alpha}$ for x variable and k fixed in the short run. For $\alpha = 1/2$, $w_x = 4$, and $w_k = 1$, the short-run profit and short-run supply functions reduce to

$$\pi^j(p, k) = p^2 k/16 - k, \tag{E.1}$$

$$q^j = pk/8. \tag{E.2}$$

With market demand of

$$q^d = 294/p \tag{E.3}$$

and 48 firms in the industry, we obtained a short-run equilibrium price of $p^* = 7$, giving firm profits of $\pi^j = 2.0625 > 0$.

In the long run, firms may enter in response to positive profits and incumbent firms are free to choose their plant size optimally. Market price will be driven to a level where maximum firm profits are zero. From (E.1), we can see that regardless of the firm's chosen plant size, this will occur only when $\hat{p} = 4$ because

$$\pi(\hat{p}, k) = k(\hat{p}^2/16 - 1) = 0 \tag{E.4}$$

for *all* $k > 0$ if and only if $\hat{p} = 4$.

The market-clearing condition with $\hat{J}$ firms, each operating a plant of size $\hat{k}$, requires that $q^d(\hat{p}) = q^s(\hat{p})$, or

$$\frac{294}{4} = \frac{4}{8}\hat{J}\hat{k}.$$

This is turn requires that

$$147 = \hat{J}\hat{k}. \tag{E.5}$$

Because at $\hat{p} = 4$ firm profits are zero regardless of plant size $\hat{k}$, long-run equilibrium is consistent with a wide range of market structures indeed. From (E.4) and (E.5), long-run equilibrium may involve a single firm operating a plant of size $\hat{k} = 147$, two firms each with plants $\hat{k} = 147/2$, three firms with plants $\hat{k} = 147/3$, all the way up to any number J of firms, each with a plant of size $147/J$. This indeterminacy in the long-run equilibrium number of firms is a phenomenon common to *all* constant-returns industries. You are asked to show this in the exercises. □

4.2 IMPERFECT COMPETITION

Perfect competition occupies one polar extreme on a spectrum of possible market structures ranging from the "more" to the "less" competitive. **Pure monopoly**, the least competitive market structure imaginable, is at the opposite extreme. In pure monopoly, there is a single seller of a product for which there are no close substitutes in consumption, and entry into the market is completely blocked by technological, financial, or legal impediments.

The monopolist takes the market demand function as given and chooses price and quantity to maximize profit. Because the highest price the monopolist can charge for any given quantity, q, is inverse demand, $p(q)$, the firm's choice can be reduced to that of choosing q, alone. The firm would then set price equal to $p(q)$.

As a function of q, profit is the difference between revenue, $r(q) = p(q)q$, and cost, $c(q)$. That is, $\Pi(q) \equiv r(q) - c(q)$. If $q^* > 0$ maximizes profit, it satisfies the first-order condition $\Pi'(q^*) \equiv r'(q^*) - c'(q^*) = 0$. This, in turn, is the same as the requirement that marginal revenue equal marginal cost:

$$mr(q^*) = mc(q^*). \tag{4.4}$$

Equilibrium price will be $p^* = p(q^*)$, where $p(q)$ is the inverse market demand function.

Let's explore the monopolist's output choice a bit further. Because $r(q) \equiv p(q)q$, differentiating to obtain marginal revenue gives

$$\begin{aligned} mr(q) &= p(q) + q\frac{dp(q)}{dq} \\ &= p(q)\left[1 + \frac{dp(q)}{dq}\frac{q}{p(q)}\right] \\ &= p(q)\left[1 - \frac{1}{|\epsilon(q)|}\right], \end{aligned} \tag{4.5}$$

where $\epsilon(q)$ is the elasticity of market demand at output q, and $|\epsilon(q)| = -(dq/dp)(p/q) > 0$ whenever market demand is negatively sloped. By combining (4.4) and (4.5), q^* will satisfy

$$p(q^*)\left[1 - \frac{1}{|\epsilon(q^*)|}\right] = mc(q^*) \geq 0 \tag{4.6}$$

because marginal cost is always nonnegative. Price is also nonnegative, so we must have

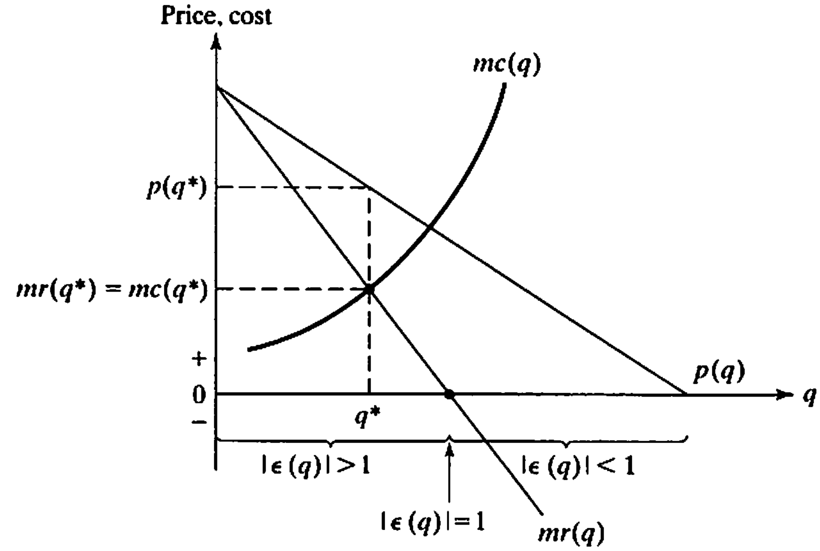

Figure 4.3. Equilibrium in a pure monopoly.

$|\epsilon(q^*)| \geq 1$. Thus, the monopolist never chooses an output in the *inelastic* range of market demand, and this is illustrated in Fig. 4.3.

Rearranging (4.6), we can obtain an expression for the percentage deviation of price from marginal cost in the monopoly equilibrium:

$$\frac{p(q^*) - mc(q^*)}{p(q^*)} = \frac{1}{|\epsilon(q^*)|}. \tag{4.7}$$

When market demand is less than infinitely elastic, $|\epsilon(q^*)|$ will be finite and the monopolist's price will exceed marginal cost in equilibrium. Moreover, price will exceed marginal cost by a greater amount the more market demand is *inelastic*, other things being equal.

As we've remarked, pure competition and pure monopoly are opposing extreme forms of market structure. Nonetheless, they share one important feature: Neither the pure competitor nor the pure monopolist needs to pay any attention to the actions of other firms in formulating its own profit-maximizing plans. The perfect competitor individually cannot affect market price, nor therefore the actions of other competitors, and so only concerns itself with the effects of its own actions on its own profits. The pure monopolist completely controls market price and output, and need not even be concerned about the possibility of entry because entry is effectively blocked.

Many markets display a blend of monopoly and competition simultaneously. Firms become more *interdependent* the smaller the number of firms in the industry, the easier entry, and the closer the substitute goods available to consumers. When firms perceive their interdependence, they have an incentive to take account of their rivals' actions and to formulate their own plans *strategically*. In Chapter 7, we'll have a great deal more to say about strategic behavior and how to analyze it, but here we can take a first look at some of the most basic issues involved.

When firms are behaving strategically, one of the first things we need to do is ask ourselves how we should characterize *equilibrium* in situations like this. On the face of it, one might be tempted to reason as follows: Because firms are aware of their interdependence, and because the actions of one firm may reduce the profits of others, won't they simply work together or *collude* to extract as much total profit as they can from the market and then divide it between themselves? After all, if they can work together to make the profit "pie" as big as possible, won't they then be able to divide the pie so that each has at least as big a slice as they could otherwise obtain? Putting the legality of such collusion aside, there is something tempting in the idea of a **collusive equilibrium** such as this. However, there is also a problem.

Let's consider a simple market consisting of J firms, each producing output q^j. We'll suppose each firm's profit is adversely affected by an increase in the output of any other firm, so that

$$\Pi^j = \Pi^j(q^1, \ldots, q^j, \ldots, q^J) \qquad \text{and} \qquad \partial\Pi^j/\partial q^k < 0, \qquad j \neq k. \tag{4.8}$$

Now suppose firms cooperate to maximize joint profits. If $\bar{\mathbf{q}}$ maximizes $\sum_{j=1}^{J} \Pi^j$, it must satisfy the first-order conditions

$$\frac{\partial\Pi^k(\bar{\mathbf{q}})}{\partial q^k} + \sum_{j\neq k} \frac{\partial\Pi^j(\bar{\mathbf{q}})}{\partial q^k} = 0, \qquad k = 1, \ldots, J. \tag{4.9}$$

Note that (4.8) and (4.9) together imply

$$\frac{\partial\Pi^k(\bar{\mathbf{q}})}{\partial q^k} > 0, \qquad k = 1, \ldots, J.$$

Think what this means. Because *each* firm's profit is increasing in its own output at $\bar{\mathbf{q}}$, each can increase its *own* profit by increasing output away from its assignment under $\bar{\mathbf{q}}$—provided, of course, that everyone else continues to produce their assignment under $\bar{\mathbf{q}}$! If even one firm succumbs to this temptation, $\bar{\mathbf{q}}$ will *not* be the output vector that prevails in the market.

Virtually all collusive solutions give rise to incentives such as these for the agents involved to cheat on the collusive agreement they fashion. Any appeal there may be in the idea of a collusive outcome as the likely "equilibrium" in a market context is therefore considerably reduced. It is perhaps more appropriate to think of self-interested firms as essentially *noncooperative*. To be compelling, any description of equilibrium in imperfectly competitive markets must take this into account.

The most common concept of noncooperative equilibrium is due to John Nash (1951). In a **Nash equilibrium**, every agent must be doing the very best he or she can, given the actions of all other agents. It is easy to see that when all agents have reached such a point, none has any incentive to change unilaterally what he or she is doing, so the situation is sensibly viewed as an equilibrium.

In a market situation like the ones we've been discussing, the agents concerned are firms. There, we will not have a Nash equilibrium until every firm is maximizing its own profit, given the profit-maximizing actions of all other firms. Clearly, the joint profit-maximizing output vector $\bar{\mathbf{q}}$ in (4.9) does not satisfy the requirements of a Nash equilibrium because, as we observed, *no* firm's individual profit is maximized at $\bar{\mathbf{q}}$ given the output choices of the other firms. Indeed, if $\mathbf{q}^*$ is to be a Nash equilibrium, each firm's output must maximize its own profit given the other firms' output choices. Thus, $\mathbf{q}^*$ must satisfy the first-order conditions:

$$\frac{\partial \Pi^k(\mathbf{q}^*)}{\partial q^k} = 0, \qquad k = 1, \ldots, J. \tag{4.10}$$

Clearly, there is a difference between (4.9) and (4.10). In general, they will determine quite different output vectors.

In what follows, we shall employ the Nash equilibrium concept in a number of different settings in which firms' decisions are interdependent.

4.2.1 COURNOT OLIGOPOLY

The following oligopoly model dates from 1838 and is due to the French economist Auguste Cournot (1838). Here we consider a simple example of **Cournot oligopoly** in the market for some homogeneous good. We'll suppose there are J identical firms, that entry by additional firms is effectively blocked, and that each firm has identical costs,

$$C(q^j) = cq^j, \qquad c \geq 0 \qquad \text{and} \qquad j = 1, \ldots, J. \tag{4.11}$$

Firms sell output on a common market, so market price depends on the total output sold by all firms in the market. Let inverse market demand be the linear form,

$$p = a - b\sum_{j=1}^{J} q^j, \tag{4.12}$$

where $a > 0$, $b > 0$, and we'll require $a > c$. From (4.11) and (4.12), profit for firm j is

$$\Pi^j(q^1, \ldots, q^J) = \left(a - b\sum_{k=1}^{J} q^k\right)q^j - cq^j. \tag{4.13}$$

We seek a vector of outputs $(\bar{q}_1, \ldots, \bar{q}_J)$ such that each firm's output choice is profit-maximizing given the output choices of the other firms. Such a vector of outputs is called a **Cournot-Nash equilibrium**. This name gives due credit to Cournot, who introduced this solution to the oligopoly problem, and to Nash, who later developed the idea more generally.

So, if $(\bar{q}_1, \ldots, \bar{q}_J)$ is a Cournot-Nash equilibrium, $\bar{q}_j$ must maximize (4.13) when $q_k = \bar{q}_k$ for all $k \neq j$. Consequently, the derivative of (4.13) with respect to q_j must be zero when $q_k = \bar{q}_k$ for all $k = 1, \ldots, J$. Thus,

$$a - 2b\bar{q}_j - b\sum_{k \neq j} \bar{q}_k - c = 0,$$

which can be rewritten

$$b\bar{q}_j = a - c - b\sum_{k=1}^{J}\bar{q}_k. \tag{4.14}$$

Noting that the right-hand side of (4.14) is independent of which firm j we are considering, we conclude that all firms must produce the same amount of output in equilibrium. By letting $\bar{q}$ denote this common equilibrium output, (4.14) reduces to $b\bar{q} = a - c - Jb\bar{q}$, which implies that

$$\bar{q} = \frac{a-c}{b(J+1)}. \tag{4.15}$$

By using (4.15), and doing a few calculations, the full set of market equilibrium values namely, firm output, total output, market price, and firm profits are as follows:

$$\begin{aligned} \bar{q}^j &= (a-c)/b(J+1), \qquad j = 1, \ldots, J, \\ \sum_{j=1}^{J}\bar{q}^j &= J(a-c)/b(J+1), \\ \bar{p} &= a - J(a-c)/(J+1) < a, \\ \bar{\Pi}^j &= (a-c)^2/(J+1)^2 b. \end{aligned}$$

Equilibrium in this Cournot oligopoly has some interesting features. We can calculate the deviation of price from marginal cost,

$$\bar{p} - c = \frac{a-c}{J+1} > 0, \tag{4.16}$$

and observe that equilibrium price will typically exceed the marginal cost of each identical firm. When $J = 1$, and that single firm is a pure monopolist, the deviation of price from marginal cost is greatest. At the other extreme, when the number of firms $J \to \infty$, (4.16) gives

$$\lim_{J\to\infty}(\bar{p} - c) = 0. \tag{4.17}$$

Equation (4.17) tells us that price will approach marginal cost as the number of competitors becomes large. Indeed, this limiting outcome corresponds precisely to what would obtain if any finite number of these firms behaved as perfect competitors. Thus, this simple model provides another interpretation of perfect competition. It suggests that perfect competition can be viewed as a limiting case of imperfect competition, as the number of firms becomes large.

4.2.2 BERTRAND OLIGOPOLY

Almost 50 years after Cournot, another French economist, Joseph Bertrand (1883), offered a different view of firm rivalry under imperfect competition. Bertrand argued it is much more natural to think of firms competing in their choice of price, rather than quantity. This small difference is enough to completely change the character of market equilibrium.

The issues involved stand out most clearly if we concentrate on rivalry between just two firms. In a simple **Bertrand duopoly**, two firms produce a homogeneous good, each has identical marginal costs $c > 0$, and no fixed cost. Though not at all crucial, for easy comparison with the Cournot case, we can again suppose that market demand is linear in total output, Q, and write

$$Q = \alpha - \beta p,$$

where p is market price.

Firms simultaneously declare the prices they will charge and they stand ready to supply all that's demanded of them at their price. Consumers buy from the cheapest source. Thus, the firm with the lowest price will serve the entire market demand at the price it has declared, whereas the firm with the highest price, if prices differ, gets no customers at all. If both firms declare the same price, then they share market demand equally, and each serves half.

Here each firm's profit clearly depends on its rival's price as well as its own. Taking firm 1 for example, for all nonnegative prices below α/β (the price at which market demand is zero), profit will be

$$\Pi^1(p^1, p^2) = \begin{cases} (p^1 - c)(\alpha - \beta p^1), & c < p^1 < p^2, \\ \frac{1}{2}(p^1 - c)(\alpha - \beta p^1), & c < p^1 = p^2, \\ 0, & \text{otherwise.} \end{cases}$$

Note that firm 1's profit is positive as long as its price exceeds marginal cost. Other things being equal, it will be largest, of course, if firm 1 has the lowest price, and only half as large if the two firms charge the same price. Its profit need never be negative, however, because the firm can always charge a price equal to marginal cost and assure itself zero profits at worst. The situation for firm 2 is symmetrical. Thus, we shall suppose that each firm i restricts attention to prices $p^i \geq c$.

What is the Nash equilibrium in this market? It may be somewhat surprising, but in the unique Nash equilibrium, both firms charge a price equal to marginal cost, and both earn zero profit. Because profit functions here are discontinuous, we cannot argue the case by differentiating and solving first-order conditions. Instead, we'll just use some common sense.

Note that because the firm with the lowest price serves the entire market, each firm has an incentive to undercut its rival. It is this effect that ultimately drives the equilibrium price down to marginal cost. We now provide the formal argument.

First, note that if each firm chooses its price equal to c, then this is a Nash equilibrium. In this case, each firm serves half the market and earns zero profits because each unit is sold at cost. Moreover, by increasing its price, a firm ceases to obtain any demand at all because the other firm's price is then strictly lower. Consequently, it is not possible to earn more than zero profits. Therefore, each firm's price choice is profit-maximizing given the other's.

Next we argue that there are no other Nash equilibria. Because each firm i chooses $p_i \geq c$, it suffices to show that here are no equilibria in which $p_i > c$ for some i. So let (p_1, p_2) be an equilibrium.

If $p_1 > c$, then because p_2 maximizes firm 2's profits given firm 1's price choice, we must have $p_2 \in (c, p_1]$, because some such choice earns firm 2 strictly positive profits, whereas all other choices earn firm 2 zero profits. Moreover, $p_2 \neq p_1$ because if firm 2 can earn positive profits by choosing $p_2 = p_1$ and splitting the market, it can earn even higher profits by choosing p_2 just slightly below p_1 and supplying the entire market at virtually the same price. Therefore,

$$p_1 > c \Longrightarrow p_2 > c \quad \text{and} \quad p_2 < p_1.$$

But by switching the roles of firms 1 and 2, an analogous argument establishes that

$$p_2 > c \Longrightarrow p_1 > c \quad \text{and} \quad p_1 < p_2.$$

Consequently, if one firm's price is above marginal cost, both prices must be above marginal cost and each firm must be strictly undercutting the other, which is impossible.

In the Bertrand model, price is driven to marginal cost by competition among just *two* firms. This is striking, and it contrasts starkly with what occurs in the Cournot model, where the difference between price and marginal cost declines only as the number of firms in the market increases.

4.2.3 MONOPOLISTIC COMPETITION

Firms in both Cournot and Bertrand oligopolies sell a homogeneous product. In **monopolistic competition**, a "relatively large" group of firms sell *differentiated* products that buyers view as close, though not perfect, substitutes for one another. Each firm therefore enjoys a limited degree of monopoly power in the market for its particular product variant, though the markets for different variants are closely related. Firms produce their products with a "similar" technology. In a monopolistically competitive group, entry occurs when a new firm introduces a previously nonexistent variant of the product.

Assume a potentially infinite number of possible product variants $j = 1, 2, \ldots$. The demand for product j depends on its own price and the prices of all other variants. We'll write demand for j as

$$q^j = q^j(\mathbf{p}), \text{ where } \partial q^j/\partial p^j < 0 \text{ and } \partial q^j/\partial p^k > 0 \text{ for } k \neq j, \tag{4.18}$$

and $\mathbf{p} = (p^1, \ldots, p^j, \ldots)$. In addition, we'll assume there is always some price $\bar{p}^j > 0$ at which demand for j is zero, regardless of the prices of the other products.

Clearly, one firm's profit depends on the prices of *all* variants; being the difference between revenue and cost:

$$\Pi^j(\mathbf{p}) = q^j(\mathbf{p})p^j - c^j(q^j(\mathbf{p})). \tag{4.19}$$

Two classes of equilibria can be distinguished in monopolistic competition: short-run and long-run. In the short run, a fixed finite number of active firms choose price to maximize profit, given the prices chosen by the others. In a long-run equilibrium, entry and exit decisions can also be made. We consider each equilibrium in turn.

Let $j = 1, \ldots, \bar{J}$ be the active firms in the short run. For simplicity, set the price "charged" by each inactive firm k to $\bar{p}^k$ to ensure that each of them produces no output. (To ease notation, we'll drop explicit mention of inactive firms for the time being.)

Now suppose $\bar{\mathbf{p}} = (\bar{p}^1, \ldots, \bar{p}^J)$ is a Nash equilibrium in the short run. If $\bar{p}^j = \bar{p}^j$, then $q^j(\bar{p}) = 0$ and firm j suffers losses equal to short-run fixed costs, $\Pi^j = -c^j(0)$. However, if $0 < \bar{p}^j < \bar{p}^j$, then firm j produces a positive output and $\bar{\mathbf{p}}$ must satisfy the first-order conditions for an interior maximum of (4.19). These can be arranged in the form

$$\frac{\partial q^j(\bar{\mathbf{p}})}{\partial p^j}\left[mr^j(q^j(\bar{\mathbf{p}})) - mc^j(\mathbf{q}^j(\bar{\mathbf{p}}))\right] = 0, \tag{4.20}$$

where we've made use of (4.5). Because $\partial q^j/\partial p^j < 0$, this reduces to the familiar requirement that price and output be chosen to equate marginal revenue and marginal cost. As usual, the monopolistic competitor may have positive, negative, or zero short-run profit.

In the long run, firms will exit the industry if their profits are negative. To analyze the long run, we'll assume that each variant has arbitrarily close substitutes that can be produced at the same cost. Under this assumption, positive long-run profits for any single firm will induce the entry of arbitrarily many firms producing close substitutes. As usual, long-run equilibrium requires there to be no incentive for entry or exit. Consequently, because of our assumption, maximum achievable profits of all firms must be negative or zero, and those of every active firm must be exactly zero.

Suppose that $\mathbf{p}^*$ is a Nash equilibrium vector of long-run prices. Then the following *two* conditions must hold for all active firms j:

$$\frac{\partial q^j(\mathbf{p}^*)}{\partial p^j}\left[mr^j(q^j(\mathbf{p}^*)) - mc^j(\mathbf{q}^j(\mathbf{p}^*))\right] = 0, \tag{4.21}$$

$$\Pi^j(q^j(\mathbf{p}^*)) = 0. \tag{4.22}$$

Both short-run and long-run equilibrium for a representative active firm are illustrated in Fig. 4.4, which shows the tangency between demand and average cost in long-run equilibrium implied by (4.21) and (4.22).

4.3 Equilibrium and Welfare

To this point, we've been concerned with questions of price and quantity determination under different market structures. We've examined the agents' incentives and circumstances under competition, monopoly, and other forms of imperfect competition, and determined the corresponding equilibrium market outcome. In this section, we'll shift our focus from "prediction" to "assessment" and ask a different sort of question. Granted that different market structures give rise to different outcomes, are there means to assess these different

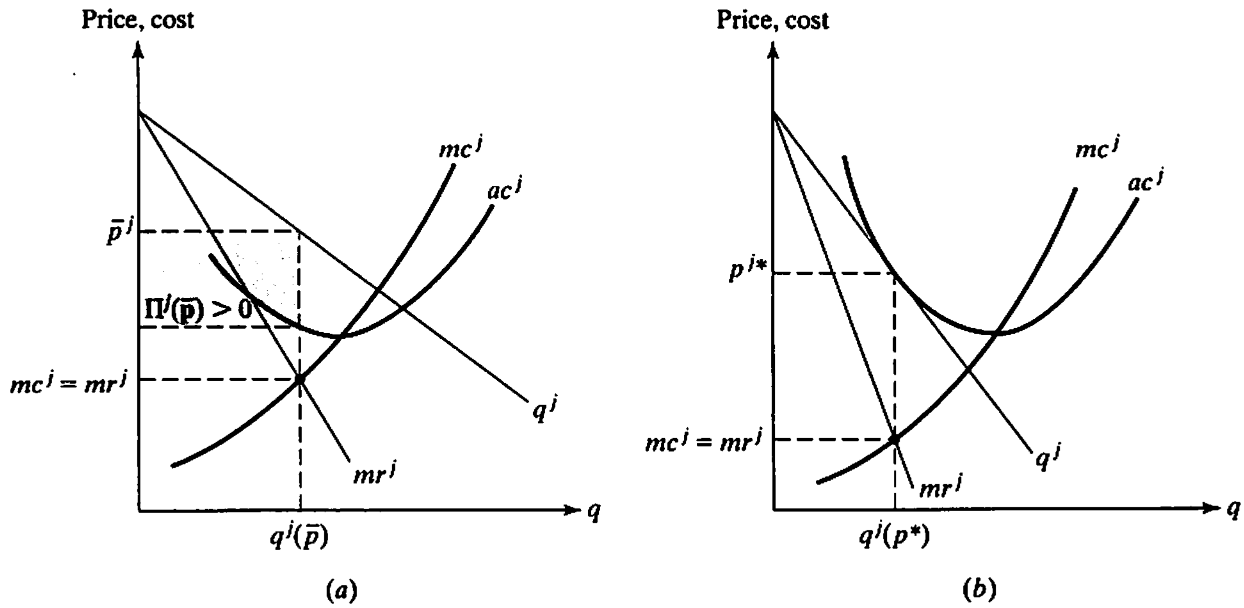

Figure 4.4. (a) Short-run and (b) long-run equilibrium in monopolistic competition.

market outcomes from a *social* point of view? Can we judge some to be "better" or "worse" than others in well-defined and meaningful ways? To answer questions like these, our focus must shift from the purely positive to the essentially normative.

Normative judgments invariably motivate and guide economic policy in matters ranging from taxation to the regulation of firms and industries. When government intervenes to change the laissez-faire market outcome, different agents will often be affected very differently. Typically, some will "win" while others will "lose." When the welfare of the individual agent is an important consideration in formulating social policy, there are really two sorts of issues involved. First, we have to ask the positive question: How will the proposed policy affect the welfare of the individual? Second, we have to ask the much more difficult normative question: How should we weigh the different effects on different individuals together and arrive at a judgment of "society's" interest? Here we'll concentrate on the first set of issues, and only dabble in the second, leaving their fuller treatment to a later chapter.

4.3.1 PRICE AND INDIVIDUAL WELFARE

It is often the case that the effect of a new policy essentially reduces to a change in prices that consumers face. Taxes and subsidies are obvious examples. To perform the kind of welfare analysis we have in mind, then, we need to know how the price of a good affects a person's welfare. To keep things simple, let's suppose the price of every other good except good q remains fixed throughout our discussion. This is the essence of the partial equilibrium approach.

So, if the price of good q is p, and the vector of all other prices is $\mathbf{p}$, then instead of writing the consumer's indirect utility as $v(p, \mathbf{p}, y)$, we shall simply write it as $v(p, y)$. Similarly, we shall suppress the vector $\mathbf{p}$ of other prices in the consumer's expenditure function, and in both her Hicksian and Marshallian demand functions. In fact, it will be convenient to introduce a **composite commodity**, m, as the amount of income spent on all goods other than q. If $\mathbf{x}(p, \mathbf{p}, y)$ denotes demand for the vector of all other goods, then the demand for the composite commodity is $m(p, \mathbf{p}, y) \equiv \mathbf{p} \cdot \mathbf{x}(p, \mathbf{p}, y)$, which we'll denote simply as $m(p, y)$. In Exercise 4.16, you are asked to show that if the consumer's utility function over all goods, $u(q, \mathbf{x})$, satisfies our standard assumptions, then the utility function over the two goods q and m, $\bar{u}(q, m) \equiv \max_{\mathbf{x}} u(q, \mathbf{x})$ subject to $\mathbf{p} \cdot \mathbf{x} \leq m$, also satisfies those assumptions. Moreover, we can use $\bar{u}$ to analyze the consumer's problem as if there were only two goods, q and m. That is, the consumer's demands for q and m, $q(p, y)$ and $m(p, y)$, respectively, solve

$$\max_{q,m} \bar{u}(q, m) \qquad \text{s.t.} \qquad pq + m \leq y,$$

and the maximized value of $\bar{u}$ is $v(p, y)$.

Consider now the following situation in which a typical practicing economist might find himself. The local government is considering plans to modernize the community's water-treatment facility. The planned renovations will improve the facility's efficiency and will result in a decrease in the price of water. The cost of the improvements will be offset by a one-time "water tax." The question is: Should the improvement be undertaken? If the preferences of the community are central, the issue reduces to this: Would consumers be willing to pay the additional tax to obtain the reduction in the price of water?

To answer this question, let's suppose our economist has water demand data for each consumer. In particular, he knows each consumer's Marshallian demand curve corresponding to his current income level. It turns out that from this, he can determine quite accurately how much each consumer would be willing to pay for the price reduction. Let's see how this is done.

Consider a particular consumer whose income is y^0. Suppose that the initial price of water is p^0 and that it will fall to p^1 as a result of the improvement project. By letting v denote the consumer's indirect utility function, $v(p^0, y^0)$ denotes his utility before the price fall and $v(p^1, y^0)$ his utility after. Now the amount of income the consumer is willing to give up for the price decrease will be just enough so that at the lower price and income levels he would be just as well off as at the initial higher price and income levels. Letting CV denote this *change* in the consumer's income that would leave him as well off after the price fall as he was before, we have

$$v(p^1, y^0 + CV) = v(p^0, y^0). \tag{4.23}$$

Note that in this example, CV is nonpositive because v is nonincreasing in p, increasing in y, and $p^1 < p^0$. CV would be nonnegative for a price increase ($p^1 > p^0$). In either case, (4.23) remains valid. This change in income, CV, required to keep a consumer's utility constant as a result of a price change, is called the **compensating variation**, and it was originally suggested by Hicks.

The idea is easily illustrated in the upper portion of Fig. 4.5, where the indifference curves are those of $\bar{u}(q, m)$. The consumer is initially at A, enjoying utility $v(p^0, y^0)$. When price falls to p^1, the consumer's demand moves to point B and utility rises to $v(p^1, y^0)$. Facing the new price p^1, this consumer's income must be reduced to $y^0 + CV$ (recall $CV < 0$ here) to return to the original utility level $v(p^0, y^0)$ at point C.

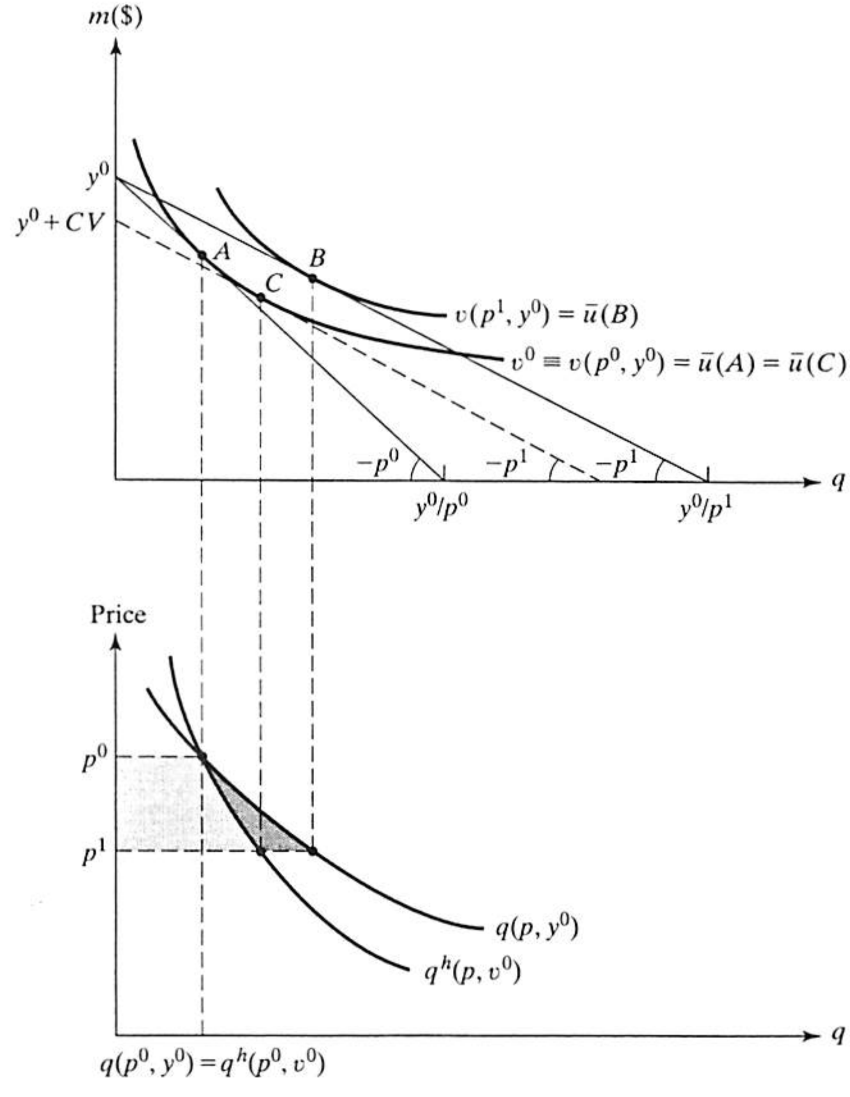

Figure 4.5. Prices, welfare, and consumer demand.

Equation (4.23) and Fig. 4.5 suggest another way to look at CV. Using the familiar identity relating indirect utility and expenditure functions, and substituting from (4.23), we must have

$$\begin{aligned} e(p^1, v(p^0, y^0)) &= e(p^1, v(p^1, y^0 + CV)) \\ &= y^0 + CV. \end{aligned} \tag{4.24}$$

Because we also know that $y^0 = e(p^0, v(p^0, y^0))$, we can substitute into (4.24), rearrange, and write

$$CV = e(p^1, v^0) - e(p^0, v^0), \tag{4.25}$$

where we've let $v^0 \equiv v(p^0, y^0)$ stand for the consumer's base utility level facing base prices and income.

Now we know that the Hicksian demand for good q is (by Shephard's lemma) given by the price partial of the expenditure function. From that and (4.25), we can write

$$\begin{aligned} CV &= e(p^1, v^0) - e(p^0, v^0) \\ &= \int_{p^0}^{p^1} \frac{\partial e(p, v^0)}{\partial p}\, dp \\ &= \int_{p^0}^{p^1} q^h(p, v^0)\, dp. \end{aligned} \tag{4.26}$$

Note then that when $p^1 < p^0$, CV is the *negative* of the area to the left of the Hicksian demand curve for base utility level v^0 between p^1 and p^0, and if $p^1 > p^0$, CV is positive and simply equal to that area. This is taken care of automatically in (4.26) because one must change the sign of the integral when the limits of integration are interchanged. In Fig. 4.5, CV is therefore equal to the (negative of the) lightly shaded area between p^0 and p^1. Study (4.26) and Fig. 4.5 carefully. You'll see, as common sense suggests, that if price rises ($p > p^0$), a positive income adjustment will be necessary to restore the original utility level ($CV > 0$), and if price declines ($p < p^0$), a negative income adjustment will restore the original utility level ($CV < 0$).

The compensating variation makes good sense as a dollar-denominated measure of the welfare impact a price change will have. Unfortunately, however, we've just learned that CV will always be the area to the left of some *Hicksian* demand curve, and Hicksian demand curves are not quite as readily observable as Marshallian ones. Of course, with enough data on the consumer's Marshallian demand system at different prices and income levels, one can recover via integrability methods the consumer's Hicksian demand and directly calculate CV. However, our economist only has access to the consumer's demand curve for this one good corresponding to one fixed level of income. And this is not generally enough information to recover Hicksian demand.

Despite this, we can still take advantage of the relation between Hicksian and Marshallian demands expressed by the Slutsky equation to obtain an estimate of CV. Recall that Marshallian demand picks up the total effect of a price change, and the Hicksian only picks up the substitution effect. The two will generally therefore diverge, and diverge precisely because of, the *income effect* of a price change. In the bottom portion of Fig. 4.5, this is illustrated for the case where q is a normal good by the horizontal deviation between the two curves everywhere but at p^0.

We'd like to relate Hick's idea of compensating variation to the notion of **consumer surplus**, because the latter is easily measured directly from Marshallian demand. Recall that at the price–income pair (p^0, y^0), consumer surplus, $CS(p^0, y^0)$, is simply the area under the demand curve (given y^0) and above the price, p^0. Consequently, the combined shaded areas in Fig. 4.5 equal the gain in consumer surplus due to the price fall from p^0 to p^1. That is,

$$\Delta CS \equiv CS(p^1, y^0) - CS(p^0, y^0) = \int_{p^1}^{p^0} q(p, y^0)\, dp. \tag{4.27}$$

$\eta \backslash a$	0.005	0.010	0.020	0.030	0.040	0.050	0.075	0.100	0.150
−1.01	−0.003	−0.005	−0.010	−0.015	−0.019	−0.024	−0.035	−0.046	0.066
0.50	0.001	0.003	0.005	0.008	0.010	0.013	0.019	0.025	0.038
0.70	0.002	0.004	0.007	0.011	0.014	0.018	0.027	0.035	0.054
0.90	0.002	0.005	0.009	0.014	0.018	0.023	0.034	0.046	0.070
1.01	0.003	0.005	0.010	0.015	0.020	0.026	0.039	0.052	0.080
1.10	0.003	0.006	0.011	0.017	0.022	0.028	0.043	0.057	0.088
1.20	0.003	0.006	0.012	0.018	0.024	0.031	0.047	0.063	0.097
1.50	0.004	0.008	0.015	0.023	0.031	0.039	0.059	0.080	0.125
2.00	0.005	0.010	0.020	0.031	0.042	0.053	0.081	0.111	0.176

Figure 4.6. Each entry gives the error as a fraction of consumer surplus, $(CV - \Delta CS)/\Delta CS$, by different values of income elasticity of demand, η, and consumer surplus as a fraction of base income, $a \equiv |\Delta CS|/y^0$. *Source*: Willig (1976).

As you can see, ΔCS will *always* be opposite in sign to CV, and it will diverge in absolute value from CV whenever demand depends in any way on the consumer's income, due to the income effect of a price change. Because we want to know CV but can only calculate ΔCS, a natural question immediately arises. How good an *approximation* of CV does ΔCS provide?

Willig (1976) studied this question and reported some useful results. He finds that when income elasticity of demand is independent of price, the absolute value of CV will diverge from that of ΔCS, but the former can be calculated exactly from knowledge of the latter.[1] More generally, for arbitrary demand functions, Willig shows that we can calculate upper and lower bounds on the size of the error we are making when we use $-\Delta CS$ as an approximation to CV. The helpful fact is this: For small price changes, the size of the error one makes when using $-\Delta CS$ instead of CV is usually so small that one can, "without apology," simply ignore it. Some of Willig's numerical results are reproduced in Fig. 4.6. Study them carefully, and remember that for a small change in the price of a single good in the consumer's budget, the proportion $a \equiv |\Delta CS|/y^0$ is likely to be very small indeed.

Let's return to our example. These approximation results suggest that as long as the price reduction from p^0 to p^1 is not too large, our economist can obtain a very good estimate indeed of each consumer's willingness to pay for it. Based on this, an informed decision can be made as to who is taxed and by how much.

Before moving on, a word of warning: When only the market demand curve, as opposed to individual demand curves, is known, the change in consumer surplus (again for small price decreases, say) will provide a good approximation to the total amount of income that consumers are willing to give up for the price decrease. However, it may well be that some of them are willing to give up more income than others (heavy water users, for example). Consequently, market demand analysis might well indicate that total willingness to pay exceeds the total cost of the project, which would imply that there is *some* way to distribute the cost of the project among consumers so that everyone is better off after paying

[1] See Exercise 4.18.

their part of the cost and enjoying the lower price. However, it would give no hint as to how that total cost should be distributed among consumers.

4.3.2 EFFICIENCY OF THE COMPETITIVE OUTCOME

In the example just considered, it seemed clear that the project should be implemented if after taking account of both the costs and benefits, everyone could be made better off. In general, when it is possible to make someone better off and no one worse off, we say that a **Pareto improvement** can be made. If there is no way at all to make a Pareto improvement, then we say that the situation is **Pareto efficient**. That is, a situation is Pareto efficient if there is no way to make someone better off without making someone else worse off.

The idea of Pareto efficiency is pervasive in economics and it is often used as one means to evaluate the performance of an economic system. The basic idea is that if an economic system is to be considered as functioning well, then given the distribution of resources it determines, it should not be possible to redistribute them in a way that results in a Pareto improvement. We shall pursue this idea more systematically in the next chapter. For now, we'll limit ourselves to the following question: Which, if any, of the three types of market competition—perfect competition, monopoly, or Cournot oligopoly—function well in the sense that they yield a Pareto-efficient outcome?

Note that the difference between the three forms of competition is simply the prices and quantities they determine. For example, were a perfectly competitive industry taken over by a monopolist, the price would rise from the perfectly competitive equilibrium price to the monopolist's profit-maximizing price and the quantity of the good produced and consumed would fall. Note, however, that in both cases, the price–quantity pair is a point on the market demand curve. The same is true of the Cournot-oligopoly solution. Consequently, we might just as well ask: Which price–quantity pairs on the market demand curve yield Pareto-efficient outcomes? We now direct our attention toward providing an answer to this question.

To simplify the discussion, we shall suppose from now on that there is just one producer and one consumer. (The arguments generalize.) Refer now to Fig. 4.7, which depicts the consumer's (and therefore the market) Marshallian demand $q(p, y^0)$, his Hicksian-compensated demand $q^h(p, v^0)$, where $v^0 = v(p^0, y^0)$, and the firm's marginal cost curve, $mc(q)$. Note then that if this firm behaved as a perfect competitor, the equilibrium price–quantity pair would be determined by the intersection of the two curves, because a competitive firm's supply curve coincides with its marginal cost curve above the minimum of its average variable costs. (We've assumed that average variable costs are minimized at $q = 0$.)

Consider now the price–quantity pair (p^0, q^0) on the consumer's demand curve above the competitive point in Fig. 4.7. We'd like to argue that this market outcome is not Pareto efficient. To do so, we need only demonstrate that we can redistribute resources in a way that makes someone better off and no one worse off.

So, consider reducing the price of q from p^0 to p^1. What would the consumer be willing to pay for this reduction? As we now know, the answer is the absolute value of the compensating variation, which, in this case, is the sum of areas A and B in the figure. Let us then reduce the price to p^1 and take $A + B$ units of income away from the consumer. Consequently, he is just as well off as he was before, and he now demands q^1 units of the good according to his Hicksian-compensated demand.

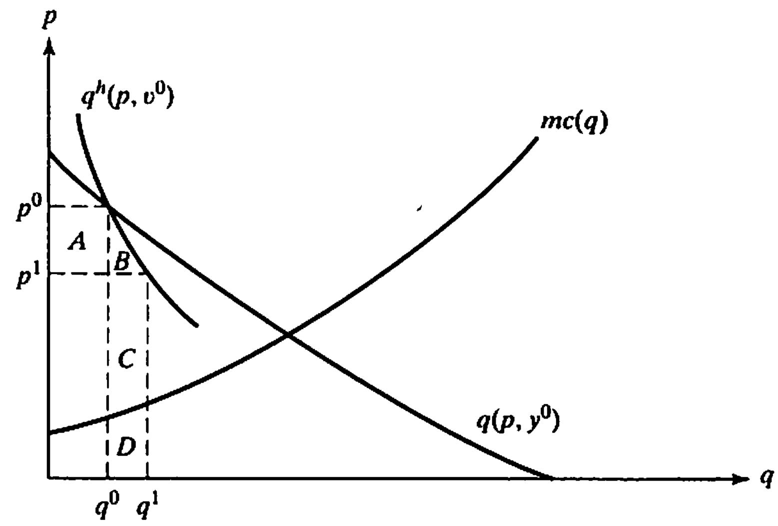

Figure 4.7. Inefficiency of monopoly equilibrium.

To fulfill the additional demand for q, let's insist that the firm produce just enough additional output to meet it.

So, up to this point, we've lowered the price to p^1, increased production to q^1, and collected $A + B$ dollars from the consumer, and the consumer is just as well off as before these changes were made. Of course, the price–quantity change will have an effect on the profits earned by the firm. In particular, if $c(q)$ denotes the cost of producing q units of output, then the change in the firm's profits will be

$$\begin{aligned}
[p^1q^1 - c(q^1)] - [p^0q^0 - c(q^0)] &= [p^1q^1 - p^0q^0] - [c(q^1) - c(q^0)] \\
&= [p^1q^1 - p^0q^0] - \int_{q^0}^{q^1} mc(q)\,dq \\
&= [C + D - A] - D \\
&= C - A.
\end{aligned}$$

Consequently, if after making these changes, we give the firm A dollars out of the $A + B$ collected from the consumer, the firm will have come out strictly ahead by C dollars. We can then give the consumer the B dollars we have left over so that in the end, *both* the consumer and the firm are strictly better off as a result of the changes we've made.

Thus, beginning from the market outcome (p^0, q^0), we have been able to make both the consumer and the firm strictly better off simply by redistributing the available resources. Consequently, the original situation was not Pareto efficient.

A similar argument applies to price–quantity pairs on the consumer's Marshallian demand curve lying below the competitive point.[2] Hence, the only price–quantity pair that can possibly result in a Pareto-efficient outcome is the perfectly competitive one—and indeed it does. We shall not give the argument here because it will follow from our more general analysis in the next chapter. However, we encourage the reader to check that the particular

[2]See Exercise 4.21.

scheme used before to obtain a Pareto improvement does not work when one begins at the competitive equilibrium. (No other scheme will produce a Pareto improvement either.)

Thus, our conclusion is that the only price–quantity pair yielding a Pareto-efficient outcome is the perfectly competitive one. In particular, neither the monopoly outcome nor the Cournot-oligopoly outcome is Pareto efficient.

Note well that we cannot conclude from this analysis that forcing a monopoly to behave differently than it would choose to must necessarily result in a Pareto improvement. It may well lower the price and increase the quantity supplied, but unless the consumers who are made better off by this change compensate the monopolist who is made worse off, the move will not be Pareto improving.

4.3.3 EFFICIENCY AND TOTAL SURPLUS MAXIMIZATION

We've seen that consumer surplus is close to being a dollar measure of the gains going to the consumer as a result of purchasing the good in question. It is easier to find an exact way to measure the dollar value to the *producer* of selling the good to the consumer. This amount, called **producer surplus**, is simply the firm's revenue over and above its variable costs.

Now it would seem that to obtain an efficient outcome, the total surplus—the sum of consumer and producer surplus—must be maximized. Otherwise, both the producer and the consumer could be made better off by redistributing resources to increase the total surplus, and then dividing the larger surplus among them so that each obtains strictly more surplus than before.

But we must take care. Consumer surplus overstates the dollar benefits to the consumer whenever income effects are present and the good is normal. Despite this, however, under the assumption that demand is downward-sloping and the firm's marginal costs are rising, efficiency will not be achieved unless the sum of consumer and producer surplus is indeed maximized.

To see this, consider again the case of a single consumer and a single producer represented in Fig. 4.8 and consider an arbitrary price-quantity pair (p, q) on the demand curve (so that $p = p(q)$, where $p(\cdot)$ is inverse demand). Earlier we defined consumer surplus at (p, q) as the area under the demand curve and above the price p. It is easy to see that we can express that same area, and so consumer surplus, as the area under the inverse demand curve up to q minus the area of the rectangle $p(q)q$. Thus, we may express the sum of consumer and producer surplus as[3]

$$
\begin{aligned}
CS + PS &= \left[\int_0^q p(\xi)\,d\xi - p(q)q\right] + [p(q)q - tvc(q)] \\
&= \int_0^q p(\xi)\,d\xi - tvc(q) \\
&= \int_0^q [p(\xi) - mc(\xi)]\,d\xi.
\end{aligned}
$$

[3]The last line follows because $\int_0^q mc(\xi)\,d\xi = c(q) - c(0)$, and $c(0)$. Because $c(0)$ is fixed cost, and $c(q)$ is total cost, the difference $c(q) - c(0)$ is total variable cost, $tvc(q)$.

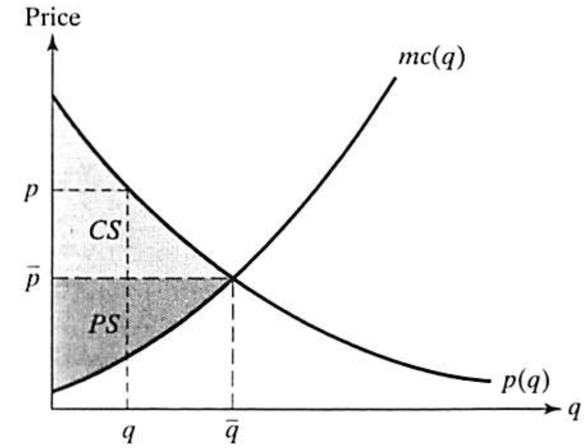

Figure 4.8. Consumer plus producer surplus is maximized at the competitive market equilibrium.

Choosing q to maximize this expression leads to the first-order condition

$$p(q) = mc(q),$$

which occurs precisely at the perfectly competitive equilibrium quantity when demand is downward-sloping and marginal costs rise, as we've depicted in Fig. 4.8.

In fact, it is this relation between price and marginal cost that is responsible for the connection between our analysis in the previous section and the present one. Whenever price and marginal cost differ, a Pareto improvement like the one employed in the previous section can be implemented. And, as we've just seen, whenever price and marginal cost differ, the total surplus can be increased.

Once again, a warning: Although Pareto efficiency requires that the total surplus be maximized, a Pareto improvement need not result simply because the total surplus has increased. Unless those who gain compensate those who lose as a result of the change, the change is not Pareto improving.

We've seen that when markets are imperfectly competitive, the market equilibrium generally involves prices that exceed marginal cost. However, "price equals marginal cost" is a necessary condition for a maximum of consumer and producer surplus. It should therefore come as no surprise that the equilibrium outcomes in most imperfectly competitive markets are *not* Pareto efficient.

EXAMPLE 4.4 Let's consider the performance of the Cournot oligopoly in Section 4.2.1. There, market demand is $p = a - bq$ for total market output q. Firms are identical, with marginal cost $c \geq 0$. When each firm produces the same output q/J, total surplus, $W \equiv cs + ps$, as a function of total output, will be

$$W(q) = \int_0^q (a - b\xi)\, d\xi - J \int_0^{q/J} c\, d\xi,$$

which reduces to

$$W(q) = aq - (b/2)q^2 - cq. \tag{E.1}$$

Because (E.1) is strictly concave, total surplus is maximized at $q^* = (a - c)/b$, where $W'(q^*) = 0$. Thus, the maximum potential surplus in this market will be

$$W(q^*) = \frac{(a-c)^2}{2b}. \tag{E.2}$$

In the Cournot-Nash equilibrium, we've seen that total market output will be $\bar{q} = J(a - c)/(J + 1)b$. Clearly, $\bar{q} < q^*$, so the Cournot oligopoly produces too little output from a social point of view. Total surplus in the Cournot equilibrium will be

$$W(\bar{q}) = \frac{(a-c)^2}{2b} \frac{J^2 + 2J}{(J+1)^2}, \tag{E.3}$$

with a **dead weight loss** of

$$W(q^*) - W(\bar{q}) = \frac{(a-c)^2}{(J+1)^2 2b} > 0. \tag{E.4}$$

By using (E.3), it is easy to show that total surplus increases as the number of firms in the market becomes larger. Before, we noted that market price converges to marginal cost as the number of firms in the oligopoly becomes large. Consequently, total surplus rises toward its maximal level in (E.2), and the dead weight loss in (E.4) declines to zero, as $J \to \infty$. □

4.4 EXERCISES

4.1 Suppose that preferences are identical and homothetic. Show that market demand for any good must be independent of the distribution of income. Also show that the elasticity of market demand with respect to the level of market income must be equal to unity.

4.2 Suppose that preferences are homothetic but not identical. Will market demand necessarily be independent of the distribution of income?

4.3 Show that if q is a normal good for every consumer, the market demand for q will be negatively sloped with respect to its own price.

4.4 Suppose that x and y are substitutes for all but one consumer. Does it follow that the market demand for x will be increasing in the price of y?

4.5 Show that the long-run equilibrium number of firms is indeterminate when all firms in the industry share the same constant returns-to-scale technology and face the same factor prices.

4.6 A firm j in a competitive industry has total cost function $c^j(q) = aq + b_j q^2$, where $a > 0$, q is firm output, and b_j is different for each firm.

(a) If $b_j > 0$ for all firms, what governs the amount produced by each of them? Will they produce equal amounts of output? Explain.

(b) What happens if $b_j < 0$ for all firms?

4.7 Technology for producing q gives rise to the cost function $c(q) = aq + bq^2$. The market demand for q is $p = \alpha - \beta q$.

(a) If $a > 0$, if $b < 0$, and if there are J firms in the industry, what is the short-run equilibrium market price and the output of a representative firm?

(b) If $a > 0$ and $b < 0$, what is the long-run equilibrium market price and number of firms? Explain.

(c) If $a > 0$ and $b > 0$, what is the long-run equilibrium market price and number of firms? Explain.

4.8 In the Cournot oligopoly of Section 4.2.1, suppose that $J = 2$. Let each duopolist have constant average and marginal costs, as before, but suppose that $0 \leq c^1 < c^2$. Show that firm 1 will have greater profits and produce a greater share of market output than firm 2 in the Nash equilibrium.

4.9 In a **Stackelberg duopoly**, one firm is a "leader" and one is a "follower." Both firms know each other's costs and market demand. The follower takes the leader's output as given and picks her own output accordingly (i.e., the follower acts like a Cournot competitor). The leader takes the follower's *reactions* as given and picks her own output accordingly. Suppose that firms 1 and 2 face market demand, $p = 100 - (q_1 + q_2)$. Firm costs are $c_1 = 10_{q_1}$ and $c_2 = q_2^2$.

(a) Calculate market price and each firm's profit assuming that firm 1 is the leader and firm 2 the follower.

(b) Do the same assuming that firm 2 is the leader and firm 1 is the follower.

(c) Given your answers in parts (a) and (b), who would firm 1 want to be the leader in the market? Who would firm 2 want to be the leader?

(d) If each firm assumes what it wants to be the case in part (c), what are the equilibrium market price and firm profits? How does this compare with the Cournot-Nash equilibrium in this market?

4.10 (Stackelberg Warfare) In the market described in Section 4.2.1, let $J = 2$.

(a) Show that if, say, firm 1 is leader and firm 2 is follower, the leader earns higher and the follower earns lower profit than they do in the Cournot equilibrium. Conclude that each would want to be the leader.

(b) If both firms decide to act as leader and each assumes the other will be a follower, can the equilibrium be determined? What will happen in this market?

4.11 In the Cournot market of Section 4.2.1, suppose that each identical firm has cost function $c(q) = k + cq$, where $k > 0$ is fixed cost.

(a) What will be the equilibrium price, market output, and firm profits with J firms in the market?

(b) With free entry and exit, what will be the long-run equilibrium number of firms in the market?

4.12 In the Bertrand duopoly of Section 4.2.2, market demand is $Q = \alpha - \beta p$, and firms have no fixed costs and identical marginal cost. Find a Bertrand equilibrium pair of prices, (p_1, p_2), and quantities, (q_1, q_2), when the following hold.

(a) Firm 1 has fixed costs $F > 0$.

(b) Both firms have fixed costs $F > 0$.

(c) Fixed costs are zero, but firm 1 has lower marginal cost than firm 2, so $c_2 > c_1 > 0$. (For this one, assume the low-cost firm captures the entire market demand whenever the firms charge equal prices.)

4.13 Duopolists producing substitute goods q_1 and q_2 face inverse demand schedules:

$$p_1 = 20 + \tfrac{1}{2}p_2 - q_1$$

and

$$p_2 = 20 + \tfrac{1}{2}p_1 - q_2,$$

respectively. Each firm has constant marginal costs of 20 and no fixed costs. Each firm is a Cournot competitor in *price*, not quantity. Compute the Cournot equilibrium in this market, giving equilibrium price and output for each good.

4.14 An industry consists of many identical firms each with cost function $c(q) = q^2 + 1$. When there are J active firms, each firm faces an identical inverse market demand $p = 10 - 15q - (J - 1)\bar{q}$ whenever an identical output of $\bar{q}$ is produced by each of the other $(J - 1)$ active firms.

(a) With J active firms, and no possibility of entry or exit, what is the short-run equilibrium output q^* of a representative firm when firms act as Cournot competitors in choosing output?

(b) How many firms will be active in the long run?

4.15 When firms $j = 1, \ldots, J$ are active in a monopolistically competitive market, firm j faces the following demand function:

$$q^j = (p^j)^{-2}\left(\sum_{\substack{i=1\\ i\neq j}}^{J} p_i^{-1/2}\right)^{-2}, \qquad j = 1, \ldots, J.$$

Active or not, each of the many firms $j = 1, 2, \ldots$ has identical costs,

$$c(q) = cq + k,$$

where $c > 0$ and $k > 0$. Each firm choses its price to maximize profits, given the prices chosen by the others.

(a) Show that each firm's demand is negatively sloped, with constant own-price elasticity, and that all goods are substitutes for each other.

(b) Show that if all firms raise their prices proportionately, the demand for any given good declines.

(c) Find the long-run Nash equilibrium number of firms.

4.16 Suppose that a consumer's utility function over all goods, $u(q, \mathbf{x})$, is continuous, strictly increasing, and strictly quasiconcave, and that the price $\mathbf{p}$ of the vector of goods, $\mathbf{x}$, is fixed. Let m denote the composite commodity $\mathbf{p} \cdot \mathbf{x}$, so that m is the amount of income spent on $\mathbf{x}$. Define the utility function $\bar{u}$ over the two goods q and m as follows.

$$\bar{u}(q, m) \equiv \max_{\mathbf{x}} \; u(q, \mathbf{x}) \text{ subject to } \mathbf{p} \cdot \mathbf{x} \leq m.$$

(a) Show that $\bar{u}(q, m)$ is strictly increasing and strictly quasiconcave. If you can, appeal to a theorem that allows you to conclude that it is also continuous.

(b) Show that if $q(p, \mathbf{p}, y)$ and $\mathbf{x}(p, \mathbf{p}, y)$ denote the consumer's Marshallian demands for q and $\mathbf{x}$, then, $q(p, \mathbf{p}, y)$ and $m(p, \mathbf{p}, y) \equiv \mathbf{p} \cdot \mathbf{x}(p, \mathbf{p}, y)$ solve

$$\max_{q,m} \; \bar{u}(q, m) \qquad \text{s.t.} \qquad pq + m \leq y.$$

and that the maximized value of $\bar{u}$ is $v(p, \mathbf{p}, y)$.

(c) Conclude that when the prices of all but one good are fixed, one can analyze the consumer's problem as if there were only two goods, the good whose price is not fixed, and the composite commodity, "money spent on all other goods."

4.17 Let $(q^0, \mathbf{x}^0) \gg \mathbf{0}$ maximize $u(q, \mathbf{x})$ subject to $p^0 q + \mathbf{p}^0 \cdot \mathbf{x} \leq y^0$. Show that if u is differentiable at $(q^0, \mathbf{x}^0)$ and $\nabla u(q^0, \mathbf{x}^0) \gg \mathbf{0}$, then the consumer would be willing to pay *strictly* more than $(p^0 - p^1)q^0$ for a reduction in the price of good q to p^1.

4.18 Willig has shown that when income elasticity of demand is independent of price, so that

$$\frac{\partial q(p, y)}{\partial y} \frac{y}{q(p, y)} \equiv \eta(y)$$

for all p and y in the relevant region, then for base price p^0 and income y^0, CS and CV are related, exactly, as follows:

$$-\Delta CS = \int_{y^0}^{CV+y^0} \exp\left(- \int_{y^0}^{\zeta} \frac{\eta(\xi)}{\xi} d\xi \right) d\zeta.$$

(a) Show that when income elasticity is constant but not equal to unity,

$$CV = y^0 \left[\frac{-\Delta CS}{y^0}(1 - \eta) + 1 \right]^{1/(1-\eta)} - y^0.$$

(b) Use this to show that when demand is independent of income, $-\Delta CS = CV$, so consumer surplus can then be used to obtain an exact measure of the welfare impact of a price change.

(c) Derive the relation between CV and ΔCS when income elasticity is unity.

(d) Finally, we can use the result in part (a) to establish a convenient rule of thumb that can be used to quickly gauge the approximate size of the deviation between the change in consumer surplus and the compensating variation when income elasticity is constant. Show that when income elasticity is constant and not equal to unity, we'll have $(CV - |\Delta CS|)/|\Delta CS| \approx (\eta|\Delta CS|)/2y^0$.

4.19 A consumer has preferences over the single good x and all other goods m represented by the utility function, $u(x, m) = \ln(x) + m$. Let the price of x be p, the price of m be unity, and let income be y.

(a) Derive the Marshallian demands for x and m.

(b) Derive the indirect utility function, $v(p, y)$.

(c) Use the Slutsky equation to decompose the effect of an own-price change on the demand for x into an income and substitution effect. Interpret your result briefly.

(d) Suppose that the price of x rises from p^0 to $p^1 > p^0$. Show that the consumer surplus area between p^0 and p' gives an *exact* measure of the effect of the price change on consumer welfare.

(e) Carefully illustrate your findings with a set of *two* diagrams: one giving the indifference curves and budget constraints on top, and the other giving the Marshallian and Hicksian demands below. Be certain that your diagrams reflect all qualitative information on preferences and demands that you've uncovered. Be sure to consider the two prices p^0 and p^1, and identify the Hicksian and Marshallian demands.

4.20 A consumer's demand for the single good x is given by $x(p, y) = y/p$, where p is the good's price, and y is the consumer's income. Let income be \$7. Find the compensating variation for an increase in the price of this good from \$1 to \$4.

4.21 Use a figure similar to Fig. 4.7 to argue that price–quantity pairs on the demand curve below the competitive price–quantity pair are not Pareto efficient.

4.22 A monopolist faces linear demand $p = \alpha - \beta q$ and has cost $C = cq + F$, where all parameters are positive, $\alpha > c$, and $(\alpha - c)^2 > 4\beta F$.

(a) Solve for the monopolist's output, price, and profits.

(b) Calculate the deadweight loss and show that it is positive.

(c) If the government requires this firm to set the price that maximizes the sum of consumer and producer surplus, and to serve all buyers at that price, what is the price the firm must charge? Show that the firm's profits are negative under this regulation, so that this form of regulation is not sustainable in the long run.

4.23 (Ramsey Rule) Building from the preceding exercise, suppose a monopolist faces negatively sloped demand, $p = p(q)$, and has costs $C = cq + F$. Now suppose that the government requires this firm to set a price (p^*) that will maximize the sum of consumer and producer surplus, subject to the constraint that firm profit be nonnegative, so that the regulation is sustainable in the long run. Show that under this form of regulation, the firm will charge a price greater than marginal cost, and that the percentage deviation of price from marginal cost ($(p^* - c)/p^*$) will be proportional to $1/\epsilon^*$, where ϵ^* is the elasticity of firm demand at the chosen price and output. Interpret your result.

4.24 Suppose that $(\bar{p}, \bar{q})$ are equilibrium market price and output in a perfectly competitive market with only two firms. Show that when demand is downward-sloping and marginal costs rise, $(\bar{p}, \bar{q})$ satisfy the second-order conditions for a maximum of consumer plus producer surplus.

4.25 (Welfare Bias in Product Selection) A monopolist must decide between two different designs for its product. Each design will have a different market demand and different costs of production. If design x_1 is introduced, it will have market demand and costs of

$$x_1 = \begin{cases} \dfrac{2}{p_1} + 6\frac{7}{8} - p_1, & \text{if } 0 < p_1 \leq 6\frac{7}{8}, \\ \dfrac{2}{p_1}, & \text{if } p_1 > 6\frac{7}{8}, \end{cases}$$

$$c_1(x_1) = 5\tfrac{1}{8} + x_1.$$

If design x_2 is introduced, it will have the following market demand and costs:

$$x_2 = 7\tfrac{7}{8} - 1\tfrac{1}{8}p_2,$$

$$c_2(x_2) = 4\tfrac{1}{8} + x_2.$$

Note that the only difference in costs between these two designs is a difference in *fixed costs*.

(a) Calculate the price the firm would charge and the profits it would make if it introduced each design. Which design will it introduce?

(b) Carefully sketch the demand and marginal cost curves for both designs on the same set of axes. Does the firm's choice maximize consumer plus producer surplus? Is the outcome Pareto efficient?

4.26 A competitive industry is in long-run equilibrium. Market demand is linear, $p = a - bQ$, where $a > 0$, $b > 0$, and Q is market output. Each firm in the industry has the same technology with cost function, $c(q) = k^2 + q^2$.

(a) What is the long-run equilibrium price? (Assume what's necessary of the parameters to ensure that this is positive and less than a.)

(b) Suppose that the government imposes a per-unit tax, $t > 0$, on every producing firm in the industry. Describe what would happen in the long run to the number of firms in the industry. What is the posttax market equilibrium price? (Again, assume whatever is necessary to ensure that this is positive and less than a.)

(c) Calculate the long-run effect of this tax on consumer surplus. Show that the deadweight loss from this tax exceeds the amount of tax revenue collected by the government in the posttax market equilibrium.

(d) Would a lump-sum tax, levied on producers and designed to raise the same amount of tax revenue, be preferred by consumers? Justify your answer.

(e) State the conditions under which a lump-sum tax, levied on *consumers* and designed to raise the same amount of revenue, would be preferred by consumers to either preceding form of tax.

4.27 A per-unit tax, $t > 0$, is levied on the output of a monopoly. The monopolist faces demand, $q = p^{-\epsilon}$, where $\epsilon > 1$, and has constant average costs. Show that the monopolist will increase price by more than the amount of the per-unit tax.

4.28 A firm under uncertainty faces gambles of the form $g = (p_1 \circ \pi_1, \ldots, p_n \circ \pi_n)$, where the π_i are profits and the p_i their probabilities of occurrence. The firm's owner has a VNM utility function over gambles in profit, and she is an expected utility maximizer. Prove that the firm's owner will always act to maximize *expected profit* if and only if she is *risk neutral*.

4.29 Consider a two-period monopoly facing the negatively sloped inverse demand function $p_t = p(q_t)$ in each period $t = 1, 2$. The firm maximizes the present discounted value of profits, $PDV = \sum_{t=0}^{1}(1+r)^{-t}\pi_t$, where $r > 0$ is the market interest rate, and π_t is period-t profit. In each of the following, assume that costs each period are increasing in that period's output and are strictly convex, and that PDV is strictly concave.

(a) If costs are $c_t = c(q_t)$ for $t = 0, 1$, show that the firm will "short-run profit maximize" in each period by choosing output to equate marginal cost and marginal revenue in each period.

(b) Now suppose that the firm can "learn by doing." Its first period costs are simply $c_0 = c_0(q_0)$. Its second-period costs, however, depend on first period output; $c_1 = c_1(q_1, q_0)$, where $\partial c_1/\partial q_0 < 0$. Does the firm still "short-run profit maximize" in each period? Why or why not? Interpret your results.