

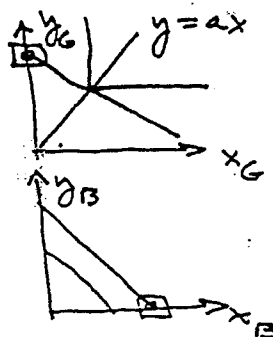
MAY 1991 MICROECONOMICS PRELIMINARY EXAM

SOLUTIONS

#31

(1) P IS THE PRICE RATIO $\frac{P_x}{P_y}$.

(a) EACH GIRL'S DEMAND: $y_G = a x_G$ AND $P x_G + y_G = 8$;
 $\therefore (a+p)x_G = 8$; $\therefore x_G = \frac{8}{a+p}$, $y_G = \frac{a}{a+p} 8$.



EACH BOY'S DEMAND:

$$x_B = \begin{cases} 8, & \text{if } p < 1 \\ [0, 8], & \text{if } p = 1 \\ 0, & \text{if } p > 1 \end{cases} \quad y_B = \begin{cases} 0, & \text{if } p < 1 \\ [0, 8], & \text{if } p = 1 \\ 8p, & \text{if } p > 1 \end{cases}$$

EXCESS DEMAND FOR HONEY ($q = \frac{P_y}{P_x}$):

IF $q < 1$ ($p > 1$): $(\frac{aq}{1+aq} 8 + \frac{8}{q} - 8)r = 8r(\frac{aq}{1+aq} + \frac{1}{q} - 1)$

IF $q > 1$ ($p < 1$): $(\frac{aq}{1+aq} 8 - 8)r = 8r(\frac{aq}{1+aq} - 1) = 8r \frac{(-1)}{1+aq}$

$\frac{aq^2 + 1 + aq - q - aq^2}{(1+aq)q}$
 $= \frac{1+aq-q}{(1+aq)q} > 0$ IF $q < 1$

EQUILIBRIUM:

IF $q < 1$: EXCESS DEMAND FOR HONEY.

IF $q > 1$: EXCESS SUPPLY OF HONEY.

IF $q = 1$: EACH BOY WILL ACCEPT ANY BUNDLE S.T.

$x_B + y_B = 8$ — e.g., $x_B = 8 - \frac{8}{1+a}$ AND $y_B = \frac{8a}{1+a}$

SO THAT $x_B + x_G = (\frac{8}{1+a} - \frac{8}{1+a} + 8) = 8r$ AND

$y_G + y_B = (\frac{a}{1+a} 8 + \frac{a}{1+a} 8)r = 8r$, CLEARING

BOTH MARKETS.

$x_B + x_G = 8 - \frac{8}{1+a} + \frac{8}{1+a} = 8$
 $y_B + y_G = \frac{8}{1+a} + \frac{a}{1+a} 8 = 8$

(b) THE PARETO OPTIMAL ALLOCATIONS ARE THE ONES THAT SATISFY

$$(1) \sum_{i=1}^r x_{Bi} + \sum_{i=1}^r x_{Gi} = 8r$$

$$(2) \sum_{i=1}^r y_{Bi} + \sum_{i=1}^r y_{Gi} = 8r$$

$$(3) \forall i: y_{Gi} = 2x_{Gi}.$$

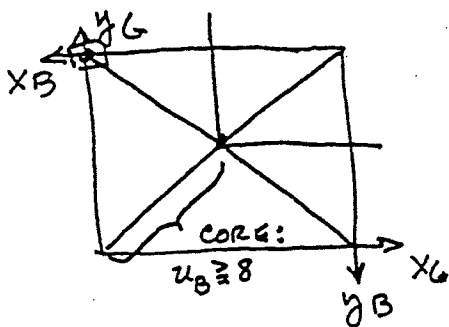
(IF $y_{Gi} > 2x_{Gi}$ FOR SOME i , SHE COULD GIVE $y_{Gi} - 2x_{Gi}$ PINTS OF HONEY TO BOYS, MAKING THEM BETTER OFF BUT NOT MAKING HER WORSE OFF. SIMILARLY IF $y_{Gi} < 2x_{Gi}$.)

IF (1), (2), (3) SATISFIED:

A GIRL'S UTILITY CAN BE ~~INCREASED~~^{INCREASED} ONLY BY GIVING HER MORE OF BOTH GOODS, CLEARLY HURTING SOMEONE ELSE.

A BOY'S UTILITY CAN BE ~~INCREASED~~^{INCREASED} ONLY BY
(i) REDUCING SOME x_{Gi} OR y_{Gi} , WHICH REDUCES u_{Gi} ; OR
(ii) REALLOCATING AMONG BOYS, WHICH CANNOT BE A PARETO IMPROVEMENT BECAUSE BOYS CARE ONLY ABOUT THE TOTAL $x_{Bi} + y_{Bi}$.

(c) $r = 1$:



THE CORE IS THE LOWER LEFT HALF OF THE MAIN DIAGONAL. ONLY THE DIAGONAL ALLOCATIONS ARE PARETO OPTIMAL W/ $r = 1$ (IN (b)); AT THEIR ENDOWMENT $u_G = 0$ AND $u_B = 8$.

$r \geq 2$: