ECON 501B FALL 2006
FIRST MIDTERM EXAM

SOLUTIONS

(1) $u_A(x,y) = \log x_A + 4\log y_A \qquad u_B(x,y) = y_B + 5\log x_B$

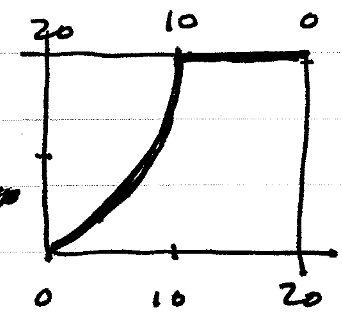
$u_{Ax} = \frac{1}{x_A},\ u_{Ay} = \frac{4}{y_A} \qquad u_{Bx} = \frac{5}{x_B},\ u_{By} = 1$

$MRS_A = \frac{1}{4}\left(\frac{y_A}{x_A}\right) \qquad MRS_B = \frac{5}{x_B}$

(a) $MRS_A = MRS_B$: $\frac{1}{4}\frac{y_A}{x_A} = \frac{5}{20-x_A}$

i.e., $\frac{1}{4} y_A = \frac{5x_A}{20-x_A}$; $y_A = \frac{20x_A}{20-x_A}$

x_A	0	4	5	8	10
y_A	0	5	$6\frac{2}{3}$	$13\frac{1}{3}$	20



FOR $x_A > 10$, $y_A = 20$ AND $x_B = 20 - x_A$, $y_B = 0$ WE HAVE

$MRS_A < \frac{1}{2}$ AND $MRS_B > \frac{1}{2}$,

$\therefore$ THESE ALLOCATIONS ARE PARETO EFFICIENT

(b) x_A: $\frac{1}{x_A} = \sigma_x$; i.e., $\frac{1}{4} = \sigma_x$

y_A: $\frac{4}{y_A} = \sigma_y$; i.e., $\frac{4}{5} = \sigma_y$

x_B: $\lambda_B \frac{5}{x_B} = \sigma_x$; i.e., $\lambda_B \frac{5}{16} = \sigma_x = \frac{1}{4}$; $\therefore \lambda_B = \frac{16}{20} = \frac{4}{5}$

y_B: $\lambda_B \cdot 1 = \sigma_y$; i.e., $\lambda_B = \frac{4}{5}$

$\therefore \lambda_B = \frac{4}{5}$, $\sigma_x = \frac{1}{4}$, $\sigma_y = \frac{4}{5}$ AND THE EQUATIONS ABOVE ESTABLISH THAT THE FOMC ARE SATISFIED.

THE COMPLEMENTARY SLACKNESS CONDITIONS:

ALL THREE LAGRANGE MULTIPLIERS ARE POSITIVE,

$\therefore$ ALL THREE CONSTRAINTS MUST BE BINDING, WHICH THEY ARE:

$x_A + x_B = 20$, $y_A + y_B = 20$, AND $u_B(16,15) = u_B(16,15)$.

(c) WE'VE SHOWN IN (b) THAT THIS INITIAL ALLOCATION IS PARETO EFFICIENT; THEREFORE IT'S ALSO A WALRASIAN EQUILIBRIUM AT PRICES PROPORTIONAL TO THE LAGRANGE VECTOR AT THIS ALLOCATION, $(\sigma_x, \sigma_y) = (\frac{1}{4}, \frac{4}{5})$. ALTERNATIVELY, THE EQUILIBRIUM PRICES ARE IN THE RATIO OF THE COMMON MRS: $\frac{p_x}{p_y} = \frac{5}{16}$. LET'S SAY THE PRICE LIST IS $(p_x, p_y) = (5, 16)$.

NOW WE'RE ASKED TO VERIFY BY DIRECT APPEAL TO THE DEFINITION THAT THE EQUILIBRIUM WE'VE IDENTIFIED IS INDEED AN EQUILIBRIUM:

AMY IS MAXIMIZING HER UTILITY:

$$MRS_A = \frac{1/4}{4/5} = \frac{5}{16} = \frac{p_x}{p_y} \text{ AND } p_x x_A + p_y y_A = (5)(4) + (16)(5) = 100. = p_x \mathring{x}_A + p_y \mathring{y}_A .$$

BEV IS MAXIMIZING HER UTILITY:

$$MRS_B = \frac{5}{16} = \frac{p_x}{p_y} \text{ AND } p_x x_B + p_y y_B = (5)(16) + (16)(5) = 160. = p_x \mathring{x}_B + p_y \mathring{y}_B .$$

AND MARKETS CLEAR:

$$x_A + x_B = 4 + 16 = \mathring{x}_A + \mathring{x}_B$$
$$y_A + y_B = 5 + 15 = \mathring{y}_A + \mathring{y}_B .$$

(d) x_A: $\frac{1}{x_A} = \sigma_x$ i.e., $\frac{1}{12} = \sigma_x$

y_A: $\frac{4}{y_A} = \sigma_y$ i.e., $\frac{4}{20} = \sigma_y$

x_B: $\lambda \frac{5}{x_B} = \sigma_x$ i.e., $\lambda \frac{5}{8} = \sigma_x$

y_B: $\lambda \cdot 1 \leqq \sigma_y$ i.e., $\frac{2}{15} \leqq 1$, OK.

$$\boxed{\begin{aligned} \sigma_x &= \tfrac{1}{12} \\ \sigma_y &= \tfrac{1}{5} \\ \lambda &= \left(\tfrac{1}{12}\right)\left(\tfrac{8}{5}\right) = \tfrac{2}{15} \end{aligned}}$$

FURTHERMORE, ALL THREE CONSTRAINTS ARE EXACTLY SATISFIED, AS THEY MUST BE WHEN ALL LAGRANGE VALUES ARE POSITIVE:

$$x_A + x_B = 12 + 8 = 20 = \mathring{x}$$
$$y_A + y_B = 20 + 0 = 20 = \mathring{y}$$
$$u_B(x_B, y_B) = 5 \log 8 = u_B(8, 0).$$