#6 ② $u^A(x_A, y_A, x_B, y_B) = 2x_A + y_A + \alpha \log x_B$.

$u^B(x_A, y_A, x_B, y_B) = x_B + y_B$. $\quad \omega_x > \alpha > 0;\ \omega_y > 0$.

MAXIMIZATION PROBLEM FOR PARETO OPTIMALITY:

$\max\ \lambda_A u^A(\cdot)$ s.t. $x_A, x_B, y_A, y_B \geqq 0$

AND TO

$$x_A + x_B \leqq \omega_x \quad : \sigma_x$$

($\lambda_A > 0$, ARBITRARILY GIVEN)

$$y_A + y_B \leqq \omega_y \quad : \sigma_y$$

$$u^B(\cdot) \geqq c \quad : \lambda_B$$

($c > 0$ ARBITRARILY GIVEN)

FIRST-ORDER CONDITIONS FOR <u>INTERIOR</u> SOLUTION:

$$\left.\begin{aligned} x_A &: \quad 2\lambda_A = \sigma_x \\ y_A &: \quad \lambda_A = \sigma_y \end{aligned}\right\} \therefore\ \sigma_x = 2\sigma_y > 0.$$

$$\left.\begin{aligned} x_B &: \quad \lambda_A \frac{\alpha}{x_B} = \sigma_x - \lambda_B \\ y_B &: \quad 0 = \sigma_y - \lambda_B \end{aligned}\right]$$

w/ $\sigma_x = 2\sigma_y$ FROM ABOVE, WE HAVE

$$\sigma_y \frac{\alpha}{x_B} = 2\sigma_y - \sigma_y;$$

i.e., $\sigma_y \dfrac{\alpha}{x_B} = \sigma_y$;

i.e., $$\boxed{\begin{aligned} &x_B = \alpha \\ &x_A = \omega_x - \alpha \\ &0 < y_A, y_B < \omega_y;\ y_A + y_B = \omega_y \end{aligned}}$$