Eco 501B Final Exam: Fall 1990

(Solutions)

#38

(1) $u^A(8,8)=64$; $u^B(4,4)=16$. Mr. A and Mr. B can unilaterally allocate their endowment (16,8). The allocations of (16,8) that are Pareto optimal for $\{A,B\}$ satisfy $MRS^A=MRS^B$ — i.e., $\frac{y_A}{x_A}=\frac{y_B}{x_B}=\frac{8}{16}=\frac{1}{2}$. The allocation $(x_A,y_A)=(11,5\frac{1}{2})$ and $(x_B,y_B)=(5,2\frac{1}{2})$ satisfies this condition, but yields $u^A=60\frac{1}{2}<64$ and $u^B=12\frac{1}{2}<16$; therefore, $\{A,B\}$ cannot improve upon the proposed allocation.

#30

(2) $MRS^A=\frac{y}{x}$ and $MRS^B=2\frac{y}{x}$.

(a) $MRS^A=2=MRS^B$; ∴ Pareto optimal if no waste. Input level is $60-(x_A+x_B)=24$; ∴ output is 48; $y_A+y_B=48$, ∴ Pareto optimal.

(b) $MRS^A=1$, $MRS^B=2$, and allocation is interior; ∴ not Pareto optimal. A Pareto-improvement: Since MC of bread is $\frac{1}{2}$ bushel, and Mr. A's marginal value of a loaf of bread is one bushel, use one of the bushels of wheat allocated to A to produce two loaves of bread, both given to A. Then $(x_A,y_A)=(19,22)$ and $u^A=418>400$.

(c) Input level is $60-(x_A+x_B)=30$; ∴ output is 60; but $y_A+y_B=50<60$, ∴ not Pareto optimal. A Pareto improvement: Give the 10 wasted loaves to someone, e.g. $(x_A\ y_A)=(20.45)$ and $(x_B\ y_B)=(10.15)$.