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(a) CAPITALIST'S PRODUCTION FUNCTION IS $f(x) = F(x, 1) = x^{2/3}$.
HE MAXIMIZES $\pi(x) = f(x) - wx$:
FOC IS $f'(x) = w$; i.e., $\frac{2}{3}x^{-\frac{1}{3}} = w$; i.e., $x^{\frac{1}{3}} = \frac{2}{3w}$.
EQUILIBRIUM REQUIRES $x = 8$ (THERE ARE 8 WORKERS FOR EVERY CAPITALIST); i.e., $2 = \frac{2}{3w}$; i.e., $w = \frac{1}{3}$.
THUS, $\pi = (8)^{2/3} - \frac{1}{3}(8) = 4 - \frac{8}{3} = \frac{4}{3}$. EACH WORKER SPENDS $\frac{1}{3}$ DOLLAR, EACH CAPITALIST SPENDS $\frac{4}{3}$ DOLLARS, FOR A TOTAL OF $(80)(\frac{1}{3}) + (10)(\frac{4}{3}) = 40$, WHICH IS THE TOTAL PRODUCTION: $10 f(x) = (10)(8)^{2/3} = 40$.

(b) WORKER'S PRODUCTION FUNCTION IS $g(y) = F(1, y) = y^{1/3}$.
HE MAXIMIZES $\pi(y) = g(y) - ry$:
FOC IS $g'(y) = r$; i.e., $\frac{1}{3}y^{-2/3} = r$; i.e., $y^{2/3} = \frac{1}{3r}$.
EQUILIBRIUM REQUIRES $y = \frac{1}{8}$; i.e., $(\frac{1}{8})^{2/3} = \frac{1}{3r}$;
i.e., $\frac{1}{4} = \frac{1}{3r}$; i.e., $r = \frac{4}{3}$. THUS, $\pi = (\frac{1}{8})^{1/3} - \frac{4}{3}(\frac{1}{8}) = \frac{1}{2} - \frac{1}{6} = \frac{1}{3}$.
EACH CAPITALIST SPENDS $\frac{4}{3}$, EACH WORKER SPENDS $\frac{1}{3}$, FOR A TOTAL OF 40 DOLLARS — JUST EXACTLY AS IN (a).

(c) THE PARETO OPTIMAL ALLOCATIONS ARE THE ONES IN WHICH THE TOTAL NUMBER OF DOLLARS TO SPEND IS MAXIMIZED AND THEN DISTRIBUTED — IN ANY WAY — TO THE 90 PEOPLE. (THEY ARE PRICE-TAKERS IN ALL MARKETS, SO THE GOODS THEY CONSUME WILL GET ALLOCATED TO THEM PARETO OPTIMALLY.) SINCE BOTH RESOURCES ARE FULLY EMPLOYED ($x = 80$, $y = 10$), THE NUMBER OF DOLLARS WILL INDEED BE MAXIMIZED, AT 40.