INTERTEMPORAL EXERCISE: SOLUTION

$u(x_0, x_1) = x_0^3 x_1^2$ $\qquad (\mathring{x}_0, \mathring{x}_1) = (38, 16)$.

$MRS = \left(\frac{3}{2}\right)\frac{x_1}{x_0}$

$$f(z) = \begin{cases} 4z - \frac{1}{8}z^2, & z \leqq 16 \\ 32, & z \geqq 16 \end{cases} \qquad f'(z) = \begin{cases} 4 - \frac{1}{4}z, & z \leqq 16 \\ 0, & z \geqq 16 \end{cases}$$

(a) $\max_z u(\mathring{x}_0 - z, \mathring{x}_1 + f(z))$

FOC: $\frac{du}{dz} \leqq 0$ AND "$=$" IF $z > 0$.

$\frac{du}{dz} = (u_0)(-1) + (u_1)(f'(z)) = 0 \iff f'(z) = \frac{u_0}{u_1}$;

i.e., $MRT = MRS$.

AT $z = 8$: $f(z) = 24$; $x_0 = 38 - 8 = 30$, $x_1 = 16 + 24 = 40$.

$MRS = \left(\frac{3}{2}\right)\frac{40}{30} = 2$, $MRT = 4 - 2 = 2$. SEE FIGURE 1.

(b) $r = 100\% = 1$: $\quad 1 + r = 2$, $\frac{1}{1+r} = \frac{1}{2}$.

THE PLANS IN (a) SATISFY $MRS = 2 = 1 + r$ AND $MRT = 2 = 1 + r$; $\therefore$ THAT'S THE INVESTMENT PLAN THAT MAXIMIZES NPV, AND THE CONSUMPTION PLAN MAXIMIZES $u(\cdot)$ AMONG THE BUNDLES WITH THE SAME NPV. SEE FIGURE 2.

THEREFORE $z = 8$, $f(z) = 24$, $x_0 = 30$, $x_1 = 40$, AND BENJAMIN NEITHER BORROWS NOR LENDS.

$V_0(38, 16) = 38 + \frac{1}{2}(16) = 46$

$V_0(-8, 24) = -8 + \frac{1}{2}(24) = 4$

$V_0(30, 40) = 30 + \frac{1}{2}(40) = 50$.

(c) $r = 200\%$: $1+r = 3$, $\frac{1}{1+r} = \frac{1}{3}$.

$$\left.\begin{aligned} V_0(38, 16) &= 38 + \tfrac{1}{3}(16) = 43\tfrac{1}{3} \\ V_0(-8, 24) &= -8 + \tfrac{1}{3}(24) = 0 \\ V_0(30, 40) &= 30 + \tfrac{1}{3}(40) = 43\tfrac{1}{3} \end{aligned}\right\} \text{(38, 16) AND (30, 40) ARE ON THE SAME NPV-CONTOUR.}$$

SEE FIGURE 3: IT'S CLEAR FROM THE DIAGRAM THAT BENJAMIN WILL CHOOSE A SMALLER z, WILL SAVE, WILL CONSUME LESS AT $t=0$ AND MORE AT $t=1$, AND WILL BE BETTER OFF (ON A HIGHER INDIFFERENCE CURVE).

(d) AT $r = 200\%$:

$f'(z) = 1+r$: $4 - \frac{1}{4}z = 3$; i.e., $z = 4$; $f(z) = 14$.

$V_0(-4, 14) = -4 + \frac{1}{3}(14) = \frac{2}{3}$

$V_0(38, 16) = 38 + \frac{1}{3}(16) = 43\frac{1}{3}$

$\therefore$ $W = V_0(38, 16) + V_0(-4, 14) = 43\frac{1}{3} + \frac{2}{3} = 44.$

$\max u(x_0, x_1)$ s.t. $x_0 + \frac{1}{1+r}x_1 = W = 44.$

SOLUTION:

$\left(\frac{3}{2}\right)\frac{x_1}{x_0} = 1 + r = 3$ AND $x_0 + \frac{1}{3}x_1 = 44$ (BC)

$\hookrightarrow$ $x_1 = 2x_0$; SUBSTITUTING INTO (BC): $x_0 + \frac{1}{3}(2x_0) = 44$

i.e., $x_0 + \frac{2}{3}x_0 = 44$

i.e., $\frac{5}{3}x_0 = 44$

i.e., $x_0 = \frac{132}{5} = 26.4$

$x_1 = 52.8$

CHECK: $\left(\frac{3}{2}\right)\left(\frac{52.8}{26.4}\right) = 3$ ✓

$26.4 + \frac{1}{3}(52.8) = 26.4 + 17.6 = 44.$ ✓

$\therefore$ $z = 4$, $f(z) = 14$; $x_0 = 26.4$, $x_1 = 52.8$;

BENJAMIN SAVES $\mathring{x}_0 - z - x_0 = 38 - 4 - 26.4 = 7.6 = s$

AND RECEIVES $(1+r)s = (3)(7.6) = 22.8$ AT $t=1$,

SO $x_1 = \mathring{x}_1 + f(z) + (1+r)s = 16 + 14 + 22.8 = 52.8.$

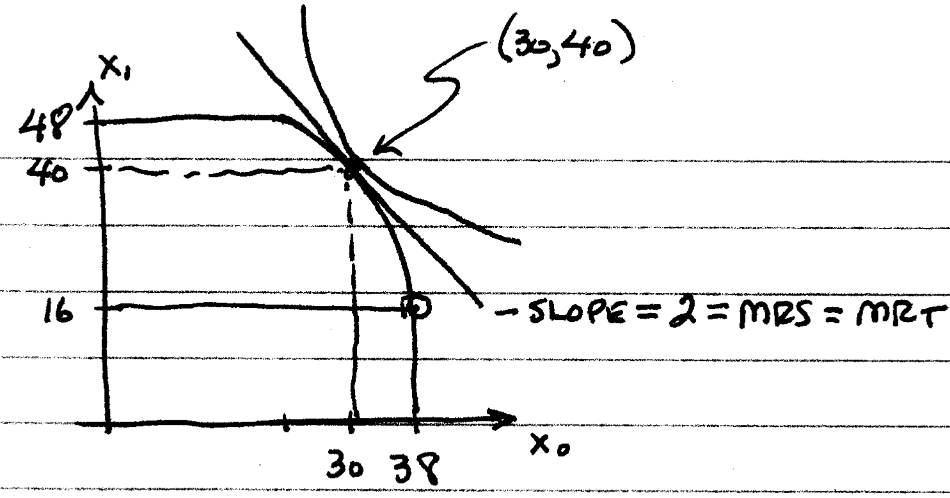

FIGURE 1

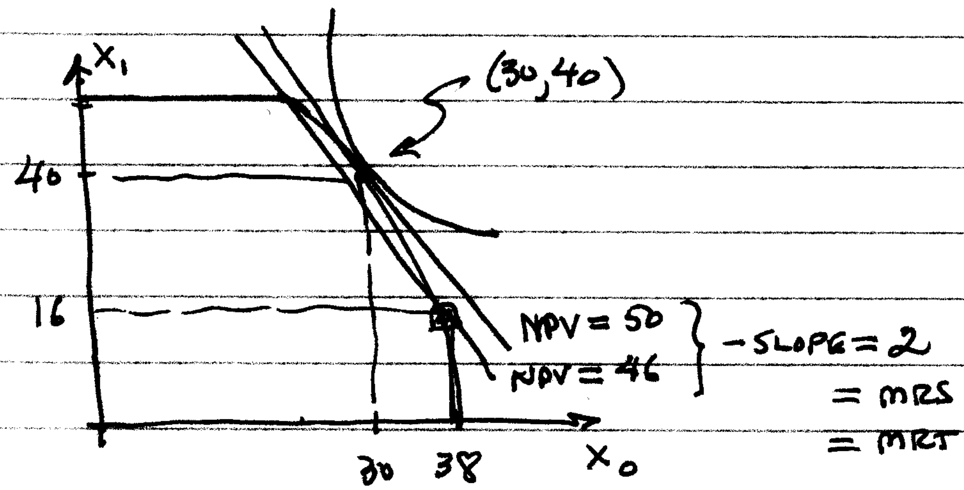

FIGURE 2

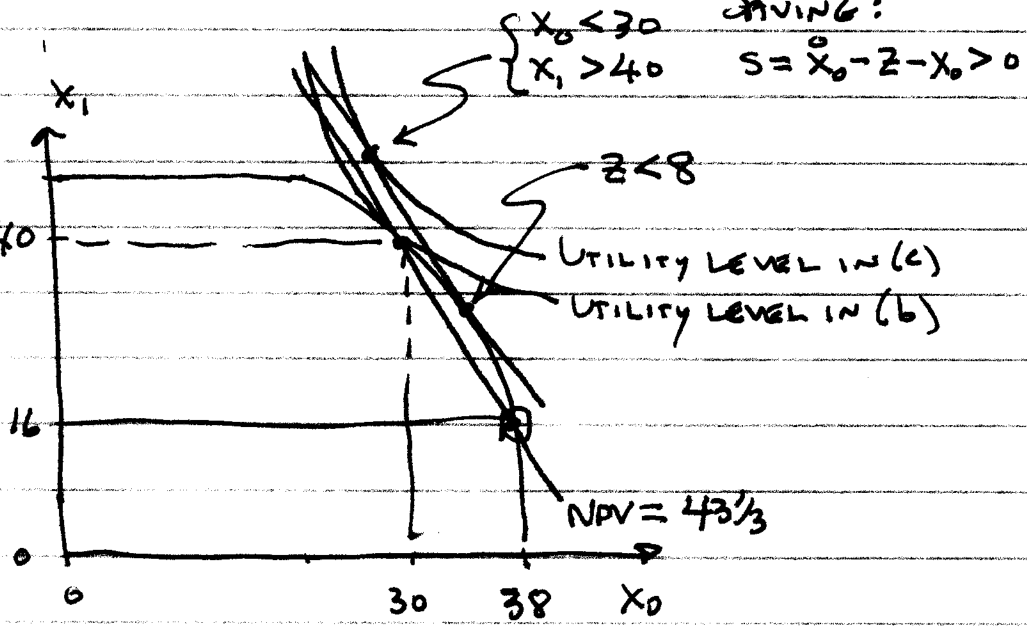

FIGURE 3