

#5.7  
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② BOTH ALLOCATIONS ARE PARETO EFFICIENT:  $MRS^i = 1$  FOR EACH  $i$ . AND BOTH ARE INDIVIDUALLY RATIONAL:  $u^i > 0 = \bar{u}^i$ . IT REMAINS TO CHECK THE TWO-PLAYER COALITIONS.

(a)  $S = \{A, C\}$  HAS ENDOWMENT  $(4, 8) = (4, 0) + (0, 8)$ . THE ALLOCATION  $(x_A, y_A) = (1, 2)$  AND  $(x_C, y_C) = (3, 6)$  IS UNILATERALLY FEASIBLE FOR  $S$ , AND IS AN IMPROVEMENT:  $u_A = 2 > 1$  AND  $u_C = 18 > 16$ .  
 $\therefore$  THE PROPOSAL IS NOT IN THE CORE.

(b)  $S = \{A, C\}$  HAS  $(4, 8)$ , AS ABOVE. THE EFFICIENT ALLOCATIONS FOR  $S$  HAVE  $MRS^i = 2$  — i.e.,  $y_i = 2x_i$ .  
 $\therefore$  THE EFFICIENT WAY TO GIVE A THE UTILITY LEVEL  $u_A = 4$ , AS IN THE PROPOSAL, IS  $(x_A)(2x_A) = 4$  — i.e.,  $x_A = \sqrt{2}$  AND  $y_A = 2\sqrt{2}$ . BUT THEN  $(x_C, y_C) = (4 - \sqrt{2}, 8 - 2\sqrt{2})$ , AND  $\therefore u_C =$

$$\begin{aligned}\therefore u_C &= (4 - \sqrt{2})(8 - 2\sqrt{2}) \\ &= 32 - 16\sqrt{2} + 4 \\ &= 36 - 16\sqrt{2} \\ &= 4[9 - 4\sqrt{2}] \\ &< 4[9 - (4)(1.4)] \\ &= 4[9 - 5.6] \\ &= (4)(3.4) < 16, \text{ THE PROPOSED UTILITY.}\end{aligned}$$

$\therefore S$  CANNOT IMPROVE. SIMILARLY  $\{B, C\}$  CANNOT IMPROVE. CLEARLY,  $\{A, B\}$  CANNOT IMPROVE, SINCE IT HAS NO HONEY.

$\therefore$  THE PROPOSAL IS IN THE CORE.