

SOLUTIONS TO 501 FINAL EXAM
SPRING 1987

#17

#17 (a) ① $P = 24 - Q$, $Q = x_1 + x_2$, $C_1(x_1) = 8x_1$, $C_2(x_2) = 4x_2$.

$$\pi_1(x_1, x_2) = (24 - x_1 - x_2)x_1 - 8x_1$$

$$\frac{\partial \pi_1}{\partial x_1} = 24 - x_2 - 2x_1 = 0 \Leftrightarrow x_1 = 12 - \frac{1}{2}x_2$$

$$\pi_2(x_1, x_2) = (24 - x_1 - x_2)x_2 - 4x_2$$

$$\frac{\partial \pi_2}{\partial x_2} = 24 - x_1 - 2x_2 = 0 \Leftrightarrow x_2 = 12 - \frac{1}{2}x_1$$

SOLVING FOC SIMULTANEOUSLY FOR EQUIL'UM:

$$\left. \begin{array}{l} 2x_1 + x_2 = 16 \\ x_1 + 2x_2 = 20 \end{array} \right\} \begin{array}{l} x_1 = 4, x_2 = 8; Q = 12, P = \$12 \\ \pi_1 = \$16, \pi_2 = \$64. \end{array}$$

(b) IF FIRMS COOPERATE: $x_1 = 0, x_2 = Q$.

$$\pi(Q) = (24 - Q)Q - 4Q$$

$$\pi'(Q) = 24 - 2Q = 0 \Leftrightarrow Q = 12; P = \$12; \pi = \$100.$$

THE \$100 WILL LIKELY BE DIVIDED BETWEEN THE FIRMS IN SUCH A WAY THAT $\pi_1 \geq \$16$ AND $\pi_2 \geq \$64$, FROM (a).

(c) COMPETITIVE OUTCOME: $P = MC = 4; x_1 = 0$ ($MC_1 > 4$).

$$Q = 24 - P = 20, \pi_1 = \pi_2 = 0.$$