

MICROECONOMICS COMP SOLUTIONS

JANUARY 1988

#18

#1 - #4 : 30 each

#5 - #9 : 10 each

PART I

#18 (1) $x = 40 - p$ $c_1(q_1, q_2) = 2(q_1 q_2 + q_1^2)$
 $p = 40 - x$ $c_2(q_1, q_2) = 3(q_1 q_2 + q_2^2)$

(a) Cournot:

$$\begin{aligned} \pi_1(q_1, q_2) &= (40 - (q_1 + q_2))q_1 - 2(q_1 q_2 + q_1^2) \\ &= 40q_1 - 3q_1 q_2 - 3q_1^2 \end{aligned}$$

$$\begin{aligned} \pi_2(q_1, q_2) &= (40 - (q_1 + q_2))q_2 - 3(q_1 q_2 + q_2^2) \\ &= 40q_2 - 4q_1 q_2 - 4q_2^2 \end{aligned}$$

$$\frac{\partial \pi_1}{\partial q_1} = 40 - 6q_1 - 3q_2 = 0 \Leftrightarrow 6q_1 + 3q_2 = 40$$

$$\frac{\partial \pi_2}{\partial q_2} = 40 - 4q_1 - 8q_2 = 0 \Leftrightarrow 4q_1 + 8q_2 = 40$$

SOLUTION: $q_1 = \frac{50}{9}, q_2 = \frac{20}{9}, p = \frac{290}{9}$

(b) COOPERATIVE:

$$\begin{aligned} C(q_1, q_2) &= c_1(q_1, q_2) + c_2(q_1, q_2) \\ &= 5q_1 q_2 + 2q_1^2 + 3q_2^2 \end{aligned}$$

COOPERATION (MAXIMIZATION OF $\pi = \pi_1 + \pi_2$)
 REQUIRES THAT $MC_1 = MC_2$ (WHERE $MC_i = \frac{\partial C}{\partial q_i}$)
 IF $q_1, q_2 > 0$. WE HAVE

$$\frac{\partial C}{\partial q_1} = 4q_1 + 5q_2 \quad \text{AND} \quad \frac{\partial C}{\partial q_2} = 5q_1 + 6q_2.$$

Clearly, if $q_1, q_2 \geq 0$, then $MC_1 \leq MC_2$,
with $MC_1 < MC_2$ if $x > 0$ (i.e., $q_1 > 0$ or $q_2 > 0$).
Thus, $q_2 = 0$ — i.e., $x = q_1$, AND WE HAVE

$$\pi(x) = (40-x)x - 2x^2 = 40x - 3x^2;$$

$$\pi'(x) = 40 - 6x; \quad \therefore \boxed{x = \frac{40}{6} = \frac{20}{3} = q_1; \quad q_2 = 0; \quad P = \frac{100}{3}.$$

(c) COMPETITIVE:

IF BOTH q_1 AND $q_2 > 0$, THEN $P = MC_1 = MC_2$ AT
COMPETITIVE EQUIL'UM, WHERE

$$MC_1 = \frac{\partial C_1}{\partial q_1} = 4q_1 + 2q_2 \quad \text{AND} \quad MC_2 = \frac{\partial C_2}{\partial q_2} = 3q_1 + 6q_2.$$

$$MC_1 = MC_2: \quad 4q_1 + 2q_2 = 3q_1 + 6q_2; \quad \text{i.e., } q_1 = 4q_2.$$

$$\text{Thus, } q_1 = \frac{4}{5}x \quad \text{AND} \quad q_2 = \frac{1}{5}x;$$

$$MC_1 = \frac{18}{5}x = MC_2; \quad \therefore P = \frac{18}{5}x;$$

$$\text{BUT ALSO } P = 40 - x; \quad \therefore 40 - x = \frac{18}{5}x;$$

$$\text{i.e., } \frac{23}{5}x = 40; \quad \text{i.e., } x = \frac{200}{23};$$

$$\therefore P = 40 - \frac{200}{23} = \frac{720}{23}.$$

$$\text{Summary: } \boxed{q_1 = \frac{160}{23}, \quad q_2 = \frac{40}{23}, \quad x = \frac{200}{23}, \quad P = \frac{720}{23}.$$