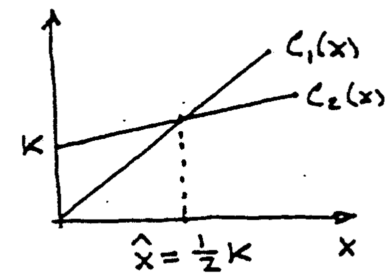
#20

~~18~~ (a) IT IS CLEAR THAT IF $X_2 > 0$ THEN $X_1 = 0$: IF BOTH X_1 AND X_2 ARE POSITIVE, EVERY UNIT SHIFTED FROM PLANT 1 TO PLANT 2 SAVES \$2. THUS, ~~~~ ALL PRODUCTION WILL BE CARRIED OUT IN JUST ONE OF THE PLANTS.



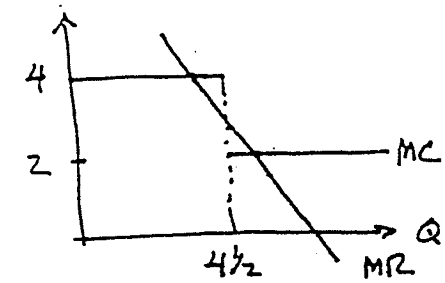
IT IS ALSO CLEAR THAT ALL IS IN PLANT 1 WHEN OUTPUT IS SMALL, ALL IN PLANT 2 WHEN OUTPUT IS LARGE. TO DETERMINE THE OUTPUT $\hat{x}$ AT WHICH THE SWITCH OCCURS, SOLVE $C_1(x) = C_2(x)$: $4x = K + 2x$; i.e., $x = \frac{1}{2}K$.

NOW WE HAVE THE FIRM'S COST FUNCTION:

$$C(Q) = \begin{cases} 4Q & \text{IF } Q \leqq \frac{1}{2}K \\ K + 2Q & \text{IF } Q \geqq \frac{1}{2}K. \end{cases} \qquad MC = \begin{cases} 4 & \text{IF } Q < \frac{1}{2}K \\ 2 & \text{IF } Q > \frac{1}{2}K. \end{cases}$$

THE FIRM'S REVENUE: $12Q - Q^2$. THUS, $MR = 12 - 2Q$.

SETTING $MR = MC$:



IF $Q < \frac{1}{2}K$: $MC = 4$, $\therefore Q = 4, p = 8$,
$\pi = 32 - 16 = 16$.

IF $Q > \frac{1}{2}K$: $MC = 2$, $\therefore Q = 5, p = 7$,
$\pi = 35 - K - 10 = 25 - K$.

SINCE $25 - K > 16 \Leftrightarrow K < 9$, WE HAVE:

IF $K < 9$, THEN $p = 7$, $Q = 5$ (ALL PRODUCED IN PLANT 2);

IF $K > 9$, THEN $p = 8$, $Q = 4$ (ALL PRODUCED IN PLANT 1).

IF $K = 9$, THEN [$p = 7, Q = 5$: PLANT 2] AND [$p = 8, Q = 4$: PLANT 1] ARE EQUALLY GOOD.

(b) $\pi_1(x_1,x_2) = (12-x_1-x_2)x_1 - 4x_1 = 8x_1 - x_2x_1 - x_1^2$

$$\frac{\partial \pi_1}{\partial x_1} = (8-x_2) - 2x_1 = 0 \Leftrightarrow x_1 = 4 - \tfrac{1}{2}x_2 = r_1(x_2).$$

$$\pi_2(x_1,x_2) = (12-x_1-x_2)x_2 - 2x_2 - K = 10x_2 - x_1x_2 - x_2^2 - K$$

$$\frac{\partial \pi_2}{\partial x_2} = (10-x_1) - 2x_2 = 0 \Leftrightarrow x_2 = 5 - \tfrac{1}{2}x_1$$

... BUT MUST ALSO HAVE $\pi_2 \geqq 0$:

$(10-x_1-x_2)x_2 > K$; IF $x_2 > 0$, THEN

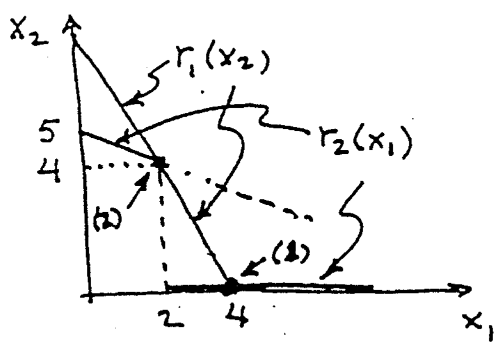
$x_2 = 5 - \frac{1}{2}x_1$; i.e., $10 - x_1 - x_2 = x_2$; $\therefore x_2^2 \geqq K$;

i.e., $x_2 \geqq \sqrt{K}$.

FIRM 2'S REACTION FUNCTION IS THUS

$$x_2 = r_2(x_1) = \begin{cases} 5 - \frac{1}{2}x_1, & \text{IF } 5 - \frac{1}{2}x_1 > \sqrt{K}; \\ 0, & \text{OTHERWISE.} \end{cases}$$

IF $K = 16$:

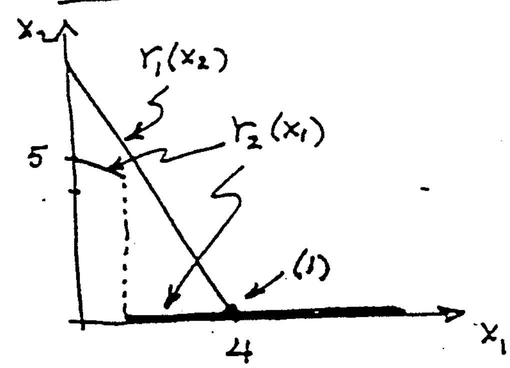


TWO NASH EQUILIBRIA:

(1) $x_1 = 4$, $x_2 = 0$, $Q = 4$,
$p = 8$, $\pi_2 = 0$,
$\pi_1 = 32 - 16 = 16$

(2) $x_1 = 2$, $x_2 = 4$, $Q = 6$,
$p = 6$, $\pi_2 = 24 - 8 - 16 = 0$,
$\pi_1 = 12 - 8 = 4$.

IF $K > 16$:



THE ONLY NASH EQUILIBRIUM IS (1): $x_1 = 4$ AND $x_2 = 0$. THE OUTCOME (2) IS NO LONGER AN EQUILIBRIUM WITH $K > 16$.

(c) IT IS CLEAR THAT A COMPETITIVE EQUILIBRIUM PRICE COULD NOT BE GREATER THAN $p = 2$ (FIRM 2 COULD INCREASE ITS PROFIT WITHOUT BOUND BY INCREASING ITS OUTPUT — BUT THIS WOULD BE AN EXCESS SUPPLY), NOR COULD IT BE SMALLER THAN $p = 2$ (NEITHER FIRM WOULD PRODUCE — AN EXCESS DEMAND). WHAT ABOUT $p = 2$? AGAIN, NEITHER FIRM WILL PRODUCE: FIRM 1 FAILS TO COVER ITS VARIABLE COST, AND FIRM 2 FAILS TO COVER ITS FIXED COST. STRICTLY SPEAKING, THEN, THERE IS NO COMPETITIVE EQUILIBRIUM.