② $u_A = y_A + 8x - \frac{1}{2}x^2$, $MRS_A = 8 - x$, IF $x \leqq 8$

$u_B = y_B + 12x - \frac{1}{2}x^2$, $MRS_B = 12 - x$, IF $x \leqq 12$

$C(x) = 4x$, $MC = 4$

(a) $MRS_A = MC$: $8 - x = 4$, $\therefore x = 4$.

(b) LET $\mathring{u}_A = \mathring{y}_A$ AND $\mathring{u}_B = \mathring{y}_B$ (i.e., UTILITIES IF $x = 0$).

AT $x = 4$:

$$u_A = \mathring{y}_A \overbrace{- 16}^{C(4)} + (8)(4) - \tfrac{1}{2}(4)^2 = \mathring{y}_A - 16 + 32 - 8 = \mathring{y}_A + 8$$

$$u_B = \mathring{y}_B + (12)(4) - \tfrac{1}{2}(4)^2 = \mathring{y}_B + 48 - 8 = \mathring{y}_B + 40$$

$\therefore$ $CS_A = 8$, $CS_B = 40$ AND $CS = 48$ IN TOTAL.

(c) MAX $\lambda_A u_A(x, y_A)$ s.t. $x, y_A, y_B \geqq 0$ AND $y_A + y_B + C(x) \leqq \mathring{y}$ AND $u_B(x, y_B) \geqq \bar{u}_B$.

FOMC (INTERIOR):

x: $\lambda_A u_{Ax} + \lambda_B u_{Bx} = \sigma C'(x)$

y_A: $\lambda_A u_{Ay} = \sigma$, i.e., $\lambda_A = \frac{\sigma}{u_{Ay}}$

y_B: $0 = \sigma - \lambda_B u_{By}$, i.e., $\lambda_B = \frac{\sigma}{u^B_y}$

COMBINING: $\sigma \frac{u_{Ax}}{u_{Ay}} + \sigma \frac{u_{Bx}}{u_{By}} = \sigma C'(x)$

i.e., $MRS_A + MRS_B = MC$.

(d) FOR ALICE AND BART: $(8 - x) + (12 - x) = 4$ IF $x \leqq 8$

i.e., $20 - 2x = 4$; $2x = 16$; $\boxed{\hat{x} = 8}$

$$\hat{u}_A = \mathring{y}_A \overbrace{- 32}^{C(8)} + (8)(8) - \tfrac{1}{2}(8)^2 = \mathring{y}_A - 32 + 64 - 32 = \mathring{y}_A \qquad CS_A = 0$$

$$\hat{u}_B = \mathring{y}_B + (12)(8) - \tfrac{1}{2}(8)^2 = \mathring{y}_B + 96 - 32 = \mathring{y}_B + 64 \qquad CS_B = 64$$

$CS = 64$.

(e) IF B PAYS t TO A TO PRODUCE AT $x=8$:

$$u_A = \hat{u}_A + t = \mathring{y}_A + t \qquad u_A \geq \mathring{u}_A \Rightarrow t \geq 8$$

$$u_B = \hat{u}_B - t = \mathring{y}_B + 64 - t \qquad u_B \geq \mathring{u}_B \Rightarrow t \leq 24.$$

(f) IF A CAN CHOOSE TO EXCLUDE B OR NOT, AND CHARGES B A PRICE p THAT MAXIMIZES A'S PROFIT:

~~B'S DEMAND IS $x = 12 - p$, i.e., $p = 12 - x$; $MR = 12 - 2x$.~~

~~$MR = MC$: $12 - 2x = 4$; $\therefore 2x = 8$; $x = 4$. $p = \$8$.~~

$\therefore R = (\$8)(4)$, $C = (\$4)(4)$, $\pi = R - C = \$16$.

$$u_A = \mathring{y}_A + \underset{\pi}{16} + 32 - 8 = \mathring{y}_A + 40 \qquad CS_A + \pi = 40$$

$$u_B = \mathring{y}_B - \underset{px}{32} + 48 - 8 = \mathring{y}_B + 8 \qquad CS_B = 8$$

TOTAL SURPLUS $= 48$

(g) A SMALL CHANGE IN p AND x WILL LEAVE π VIRTUALLY UNCHANGED, SINCE THE CURRENT p AND x MAXIMIZE π. BUT $MRS_A = 4$ AT $x = 4$, SO AN INCREASE IN x WILL INCREASE u_A. A DOES NOT NEED TO COMPARE MRS_A TO MC BECAUSE COST (AND MC) IS ALREADY ACCOUNTED FOR IN THE π-CALCULATION.

FOR SMALL CHANGES IN X

SEE THE FOLLOWING PAGE FOR THE PRODUCTION LEVEL x AND PRICE p THAT MAXIMIZE ALICE'S UTILITY. IT'S QUITE STRAIGHTFORWARD, BUT BECAUSE x AND p ARE NOT INTEGERS IT'S A BIT MESSY TO CALCULATE REVENUE, PROFIT, SURPLUS, ETC.