(3) $u_A(x_0, x_H, x_L) = x_0 + 5x_H - .3x_H^2 + 3x_L - .3x_L^2$

$u_B(x_0, x_H, x_L) = x_0 + 5x_H - .4x_H^2 + 3x_L - .2x_L^2$

$\mathring{x}_A = (6, 4, 2)$, $\mathring{x}_B = (6, 10, 8)$; $\therefore \mathring{x} = (12, 14, 10)$.

$MRS_{AH} = 5 - .6x_{AH}$ $\qquad MRS_{AL} = 3 - .6x_{AL}$

$MRS_{BH} = 5 - .8x_{BH}$ $\qquad MRS_{BL} = 3 - .4x_{BL}$

(a) $MRS_{AH} = MRS_{BH}$: $5 - .6x_{AH} = 5 - .8x_{BH}$ $\quad \therefore 6x_{AH} = 8x_{BH}$

i.e., $x_{BH} = \frac{3}{4}x_{AH}$ $\quad \therefore \boxed{x_{AH} = 8, \; x_{BH} = 6.}$

$MRS_{AL} = MRS_{BL}$: $3 - .6x_{AL} = 3 - .4x_{BL}$ $\quad \therefore 6x_{AL} = 4x_{BL}$

i.e., $x_{AL} = \frac{2}{3}x_{BL}$ $\quad \therefore \boxed{x_{AL} = 4, \; x_{BL} = 6}$

x_{A0}, x_{B0} NEED ONLY SATISFY $x_{A0} + x_{B0} = \mathring{x}_0 = 12$.

NOTE THAT $MRS_{AH} = MRS_{BH} = 5 - 4.8 = .2$

AND $MRS_{AL} = MRS_{BL} = 3 - 2.4 = .6$.

(b) THE ARROW-DEBREU EQUILIBRIUM SATISFIES $MRS_{iH} = p_H$ AND $MRS_{iL} = p_L$ FOR $i = A, B$ (ASSUMING $p_0 = 1$), SO THE RESULTS IN (a) YIELD $\boxed{p_H = .2 \text{ AND } p_L = .6.}$

THE RESULTS IN (a) THEN ALSO YIELD THE INDIVIDUALS' CHOICES OF x_{iH} AND x_{iL}: $(x_{AH}, x_{AL}) = (8, 4)$ AND $(x_{BH}, x_{BL}) = (6, 6)$.

THESE CHOICES, ALONG WITH THE PRICES, YIELD

$$x_{A0} = \mathring{x}_{A0} - p_H x_{AH} - p_L x_{AL} = 6 - .2(8-4) - .6(4-2) = 6 - .8 - 1.2 = 4$$

$$x_{B0} = \mathring{x}_{B0} - p_H x_{BH} - p_L x_{BL} = 6 - .2(6-10) - .6(6-8) = 6 + .8 + 1.2 = 8.$$

(c) LET S_i DENOTE i'S SAVING. EQUILIBRIUM SATISFIES $S_A + S_B = 0$ AND $MRS_{iH} + MRS_{iL} = \frac{1}{1+r}$ FOR EACH i (THIS IS DERIVED BELOW, ALTHOUGH IT WAS NOT REQUIRED). NOTE THAT $x_{AH} = \mathring{x}_{AH} + (1+r)S_A$ AND $x_{AL} = \mathring{x}_{AL} + (1+r)S_A$, AND SIMILARLY FOR B. WRITING z_i FOR $(1+r)S_i$ TO SIMPLIFY NOTATION, WE HAVE

$$MRS_{AH} + MRS_{AL} = 5 - .6(4+z_A) - .6(2+z_A) + 3$$
$$= 8 - 2.4 - 1.2 - 1.2z_A = 4.4 - 1.2z_A$$
$$MRS_{BH} + MRS_{BL} = 5 - .8(10+z_B) + 3 - .4(8+z_B)$$
$$= 8 - 8 - 3.2 - 1.2z_B = -3.2 - 1.2z_B$$

AT EQUILIBRIUM WE HAVE $S_B = -S_A$ (AND $\therefore z_B = -z_A$) AND

$$MRS_{AH} + MRS_{AL} = \frac{1}{1+r} = MRS_{BH} + MRS_{BL}$$

i.e., $4.4 - 1.2z_A = -3.2 - 1.2(-z_A)$

i.e., $2.4z_A = 7.6$; i.e., $z_A = \frac{76}{24} = \frac{19}{6} = 3\frac{1}{6}$.

$\therefore MRS_{AH} + MRS_{AL} = 4.4 - (1.2)\left(\frac{19}{6}\right) = 4.4 - \left(\frac{6}{5}\right)\left(\frac{19}{6}\right) = 4.4 - \frac{19}{5} = .6$.

$MRS_{BH} + MRS_{BL} = -3.2 + 1.2z_A = -3.2 + \left(\frac{6}{5}\right)\left(\frac{19}{6}\right) = -3.2 + \frac{19}{5} = .6$.

$\therefore \frac{1}{1+r} = .6$; $1+r = \frac{1}{.6} = \frac{5}{3} = 1\frac{2}{3}$; $\boxed{r = 2/3}$

SOLVING FOR S_A AND S_B:

$(1+r)S_A = z_A$; i.e., $\frac{5}{3}S_A = \frac{19}{6}$; $\therefore S_A = \left(\frac{3}{5}\right)\left(\frac{19}{6}\right) = \left(\frac{6}{10}\right)\left(\frac{19}{6}\right) = 1.9$.

$\boxed{S_A = 1.9,\ S_B = -1.9}$

$$\left.\begin{array}{l} x_{AH} = 4 + z_A = 7\frac{1}{6},\quad x_{AL} = 2 + z_A = 5\frac{1}{6} \\ x_{BH} = 10 + z_B = 6\frac{5}{6},\quad x_{BL} = 8 + z_B = 4\frac{5}{6} \end{array}\right\}$$ NOTE HOW THESE COMPARE WITH (a)!

$x_{A0} = \mathring{x}_{A0} - S_A = 6 - 1.9 = 4.1$, $x_{B0} = \mathring{x}_{B0} - S_B = 6 + 1.9 = 7.9$.

(d). TWO SECURITIES:

$d_1 = \begin{bmatrix} 1+r \\ 1+r \end{bmatrix}$, PRICE IS 1; $d_2 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, PRICE IS p.

NOTE THAT WE COULD INSTEAD HAVE DEFINED d_1 BY $d_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, WITH PRICE q. THEN THE INTEREST RATE r WOULD BE DEFINED BY $q = \frac{1}{1+r}$.

SINCE THE SECURITIES SPAN THE SPACE OF STATE-CONTINGENT RETURNS, R^2, THE EQUILIBRIUM ALLOCATION HERE WILL BE THE ARROW-DEBREU ALLOCATION, AND THE EQUILIBRIUM r AND p WILL BE RELATED TO THE ARROW-DEBREU PRICES p_H AND p_L AS FOLLOWS:

PRICE OF d_1: $1 = (1+r)p_H + (1+r)p_L = (1+r)(p_H+p_L) = (1+r)(.8)$

PRICE OF d_2: $p = (1)p_H + (0)p_L = p_H = .2$.

FROM THE EQUATION FOR d_1'S PRICE WE HAVE $\frac{1}{1+r} = .8$; $\therefore r = 1/4$.

SO WE HAVE: $\boxed{\text{INTEREST RATE IS } r = 1/4 = 25\%, \text{ PRICE OF INSURANCE IS } p = .2.}$

IF WE HAD INSTEAD DEFINED d_1 AS $d_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, WITH PRICE q, WE WOULD HAVE $q = (1)p_H + (1)p_L = .2 + .6 = .8$, SO WE AGAIN HAVE $\frac{1}{1+r} = .8$ AND THEREFORE $r = 1/4$.

THE EASIEST WAY TO DETERMINE THE SAVING AND INSURANCE CHOICES IS TO USE THE ARROW-DEBREU QUANTITIES, AS FOLLOWS:

SAVING: THE ONLY WAY TO AUGMENT x_L IS VIA SAVING; i.e., $x_L = \mathring{x}_L + (1+r)s$, OR $s = \frac{1}{1+r}(x_L - \mathring{x}_L)$.

SAVING BY A: $s_A = \frac{1}{1+r}(x_{AL} - \mathring{x}_{AL}) = .8(4-2) = 1.6$.

SAVING BY B: $s_B = \frac{1}{1+r}(x_{BL} - \mathring{x}_{BL}) = .8(6-8) = -1.6$.

THUS, A SAVES 1.6, WHICH YIELDS 2 TOMORROW IN EACH STATE; B BORROWS 1.6, REPAYING 2 TOMORROW IN EACH STATE.

INSURANCE: WITH NO INSURANCE, CONSUMPTION IN STATE H WOULD BE $x_H = \mathring{x}_H + (1+r)s$, THE SAME AS IN STATE L. BUT INSURANCE AUGMENTS THIS: BUYING ξ UNITS OF INSURANCE PROVIDES $x_H = \mathring{x}_H + (1+r)s + \xi$ — i.e., $\xi = x_H - (\mathring{x}_H + (1+r)s)$.

INSURANCE BY A:
$$\xi_A = x_{AH} - (\mathring{x}_{AH} + (1+r)s_A)$$
$$= 8 - (4+2) = 8 - 6 = 2$$

INSURANCE BY B:
$$\xi_B = x_{BH} - (\mathring{x}_{BH} + (1+r)s_B)$$
$$= 6 - (10-2) = 6 - 8 = -2.$$

THUS, A BUYS 2 UNITS OF INSURANCE, WHICH ARE SOLD BY B, AT PRICE $p = .2$.

CHECK x_{A0} AND x_{B0} AGAINST ARROW-DEBREU VALUES:
$$x_{A0} = \mathring{x}_{A0} - s_A - p\xi_A = 6 - 1.6 - (.2)(2) = 6 - 2 = 4$$
$$x_{B0} = \mathring{x}_{B0} - s_B - p\xi_B = 6 - (-1.6) - (.2)(-2) = 6 + 2 = 8,$$
THE SAME AS THE ARROW-DEBREU VALUES.

(e) THE ALLOCATION IN (d) IS PARETO EFFICIENT, THE ONE IN (c) IS NOT: THE ONE IN (d) SATISFIES THE EQUATIONS IN (a), THE ONE IN (c) DOES NOT. MORE GENERALLY, THE EXPLANATION IS THAT THERE ARE COMPLETE SECURITIES MARKETS IN (d) — THE SECURITIES SPAN THE SPACE OF STATE-CONTINGENT RETURNS, SO THAT INDIVIDUALS CAN ACHIEVE INDEPENDENT CHOICES OF x_H AND x_L. IN (c) THAT'S NOT POSSIBLE: $x_H - \mathring{x}_H$ AND $x_L - \mathring{x}_L$ MUST BE THE SAME, NAMELY $(1+r)s$.

DERIVATION OF THE MARGINAL CONDITION FOR UTILITY MAXIMIZATION WITH ONLY A CREDIT MARKET, AS IN (c):
[THIS WAS NOT REQUIRED]

DEFINE $\tilde{u}(s) := u(x_0(s), x_H(s), x_L(s))$

$$= u(\mathring{x}_0 - s,\ \mathring{x}_H + (1+r)s,\ \mathring{x}_L + (1+r)s).$$

INTERIOR FOC: $\frac{\partial \tilde{u}}{\partial s} = 0$;

i.e., $(-1)u_0 + (1+r)u_H + (1+r)u_L = 0$

i.e., $(1+r)(u_H + u_L) = u_0$

i.e., $\frac{u_H}{u_0} + \frac{u_L}{u_0} = \frac{1}{1+r}$

i.e., $MRS_H + MRS_L = \frac{1}{1+r}$.