#9.6

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$\overset{\circ}{x}^A = (5, 16, 16)$, $\overset{\circ}{x}^B = (15, 4, 4)$, $\therefore\ \overset{\circ}{x} = (20, 20, 20)$.

(a) ARROW-DEBREU EQUIL'M:

$$MRS^A_R = MRS^B_R = p_R; \quad \text{i.e., } \frac{10}{2x^A_R} = \frac{10}{3(20-x^A_R)};$$

$$\text{i.e., } 2x^A_R + 3x^A_R = 60; \quad \text{i.e., } \boxed{x^A_R = 12,\ x^B_R = 8,\ p_R = \tfrac{5}{12}}.$$

$$MRS^A_D = MRS^B_D = p_D; \quad \text{i.e., } \frac{10}{3x^A_D} = \frac{10}{2(20-x^A_D)};$$

$$\text{i.e., } 3x^A_D + 2x^A_D = 40; \quad \text{i.e., } \boxed{x^A_D = 8,\ x^B_D = 12,\ p_D = \tfrac{5}{12}}.$$

NOTE THAT $x^A_0 = 5 - (\frac{5}{12})(-8) - (\frac{5}{12})(-4) = 5 + \frac{60}{12} = 10$,
AND SIMILARLY $x^B_0 = 15 - \frac{60}{12} = 10$. [THIS ASSUMES $p_0 = 1$.]

(b) $MRS^A_R + MRS^A_D = MRS^B_R + MRS^B_D = \frac{1}{1+r}$,
AND $x^A_R = x^A_D = 16 + (1+r)y^A = 16 + \rho y^A$ ← $\rho := 1+r$
AND $x^B_R = x^B_D = 4 + (1+r)y^B = 4 + \rho y^B$, AND $y^A + y^B = 0$.

THUS, $$\left(\frac{1}{2}\right)\frac{10}{16+\rho y^A} + \left(\frac{1}{3}\right)\frac{10}{16+\rho y^A} = \left(\frac{1}{3}\right)\frac{10}{4-\rho y^A} + \left(\frac{1}{2}\right)\frac{10}{4-\rho y^A};$$

$$\text{i.e., } 16 + \rho y^A = 4 - \rho y^A; \quad \text{i.e., } \rho y^A = -6;$$

$$\therefore\ \boxed{x^A_R = x^A_D = 10 \text{ AND } x^B_R = x^B_D = 10} \text{ AND}$$

$$\frac{1}{1+r} = \frac{10}{(2)(10)} + \frac{10}{(3)(10)} = \frac{1}{2} + \frac{1}{3} = \frac{5}{6}; \quad \therefore\ \boxed{r = \tfrac{1}{5} = 20\%}$$

NOTE THAT $y^A = -5$ AND $y^B = 5$,

(c) THE (b) OUTCOME IS <u>NOT</u> PARETO EFFICIENT:

$$MRS^A_R = \frac{1}{2} > \frac{1}{3} = MRS^B_R \quad \text{AND} \quad MRS^A_D = \frac{1}{3} < \frac{1}{2} = MRS^B_D.$$

$\therefore$ THERE IS A PARETO IMPROVING TRADE AT A ONE-FOR-ONE RATE — i.e., $\Delta x^i_R = -\Delta x^i_D$, AS IN (a).

(d) $x_0 + \sum_1^4 q_i y_i \leqq \overset{\circ}{x} = 5$, TODAY

$$\begin{bmatrix} x_R \\ x_D \end{bmatrix} = y_1 \begin{bmatrix} 1 \\ 0 \end{bmatrix} + y_2 \begin{bmatrix} 0 \\ 1 \end{bmatrix} + y_3 \begin{bmatrix} 12 \\ 12 \end{bmatrix} + y_4 \begin{bmatrix} 60 \\ -12 \end{bmatrix} + \begin{bmatrix} 16 \\ 16 \end{bmatrix}.$$

(e) THESE SECURITIES SPAN THE TWO-DIMENSIONAL SPACE OF STATE-CONTINGENT RETURNS — IN PARTICULAR, IF $y_3 = y_4 = 0$, THEN THE OUTCOME IN (a) IS AVAILABLE, AT ARROW-DEBREU PRICES $p_R = p_D = \frac{5}{12}$. CONSEQUENTLY, EACH SECURITY'S PRICE MUST BE GIVEN BY

(*) $q_i = d_{Ri} p_R + d_{Di} p_D$ —

IN PARTICULAR,

$$\boxed{q_1 = p_R = \frac{5}{12}, \quad q_2 = p_D = \frac{5}{12}, \quad q_3 = 12(p_R + p_D) = \cancel{\$}10, \quad q_4 = 60p_R - 12p_D = 25 - 5 = 20.}$$

THIS IS BECAUSE, IF SOME SECURITY DOES NOT SATISFY (*), THEN AN ARBITRAGE IS POSSIBLE USING THAT SECURITY.

THE OUTCOME HERE WILL BE THE SAME AS IN (a):

$$\boxed{x^A = (10, 12, 8) \quad \text{AND} \quad x^B = (10, 8, 12).}$$