Economics 519 Midterm Exam
University of Arizona
Fall 2016

Closed-book part: Don't consult books or notes until you've handed in your solutions to Problems #1 - #3. After you do consult books or notes, your solutions to Problems #1 - #3 will not be accepted.

1. Let $\mathcal{F}$ be the set of all real functions $f : \mathbb{R} \to \mathbb{R}$.

(a) Write down a definition of addition and scalar multiplication of functions in $\mathcal{F}$ under which $\mathcal{F}$ is a vector space. You don't need to verify that this is a vector space.

(b) Let $\mathcal{P}$ be the set of all polynomial functions: a function is in $\mathcal{P}$ if and only if it can be written as $f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_2 x^2 + a_1 x + a_0$ for some $n \in \mathbb{N}$ and some $a_0, a_1, \ldots, a_n \in \mathbb{R}$. Prove that $\mathcal{P}$ is a vector subspace of $\mathcal{F}$. (Note that two functions in $\mathcal{P}$ need not be polynomials of the same degree.) You can again use any theorems we've stated in the course.

2. Let $f : \mathbb{R}^2_{++} \to \mathbb{R}$ be defined by $f(x_1, x_2) = \dfrac{1}{x_1} + \dfrac{1}{x_2}$. Use the second-order conditions to determine whether f is concave or convex, and if it's neither (or you can't tell), use second-order conditions to determine whether it's quasiconcave or quasiconvex.

3. Let $f : S \to \mathbb{R}$ be a function defined on a convex set $S \subseteq \mathbb{R}^n$; let $a, b \in \mathbb{R}$, with $a > 0$; and define $g : S \to \mathbb{R}$ by $g(x) = af(x) + b$. Using only the definition of a concave function — don't assume that the functions are differentiable — prove that g is concave if and only if f is concave.

Open-book part: Be sure to turn in your solutions to Problems #1 - #3 before using notes.

4. Let $f : \mathbb{R}^2 \to \mathbb{R}$ be defined by $f(x_1, x_2) = \frac{1}{2}x_1^2 - \frac{1}{2}x_2^2 + x_1 x_2 - x_1 + x_2$.

(a) Determine the gradient and the Hessian matrix of f as functions of (x_1, x_2).

(b) Find the critical point of f (there is only one; denote it by $\overline{\mathbf{x}} = (\overline{x}_1, \overline{x}_2)$).

(c) Write down the second-degree Taylor polynomial $P_2(\cdot)$ for f at $\overline{\mathbf{x}}$.

(d) Use the second-order conditions to determine whether $\overline{\mathbf{x}}$ is a local maximum or minimum for f.

(e) Use the first- and second-order conditions to determine the local and global maxima and minima of f (if any) subject to the constraint $x_1 + x_2 = 2$. Determine the value of the Lagrange multiplier λ and draw a diagram depicting the constraint, the critical point(s), and the gradients ∇f and ∇G, where $G : \mathbb{R}^2 \to \mathbb{R}$ is the function $G(x_1, x_2) = x_1 + x_2 = 2$.

(f) Use the first- and second-order conditions to determine the local and global maxima and minima of f (if any) subject to the constraint $x_1 - x_2 = 2$. Determine the value of the Lagrange multiplier λ and draw a diagram depicting the constraint, the critical point(s), and the gradients ∇f and ∇G, where $G : \mathbb{R}^2 \to \mathbb{R}$ is the function $G(x_1, x_2) = x_1 - x_2 = 2$.